21st Alert Doctoral School

Mathematical modelling in geomechanics
In memory of Professor Ioannis Vardoulakis

# THREE-DIMENSIONAL COSSERAT CONTINUUM MODELING OF FRACTURED ROCK MASSES

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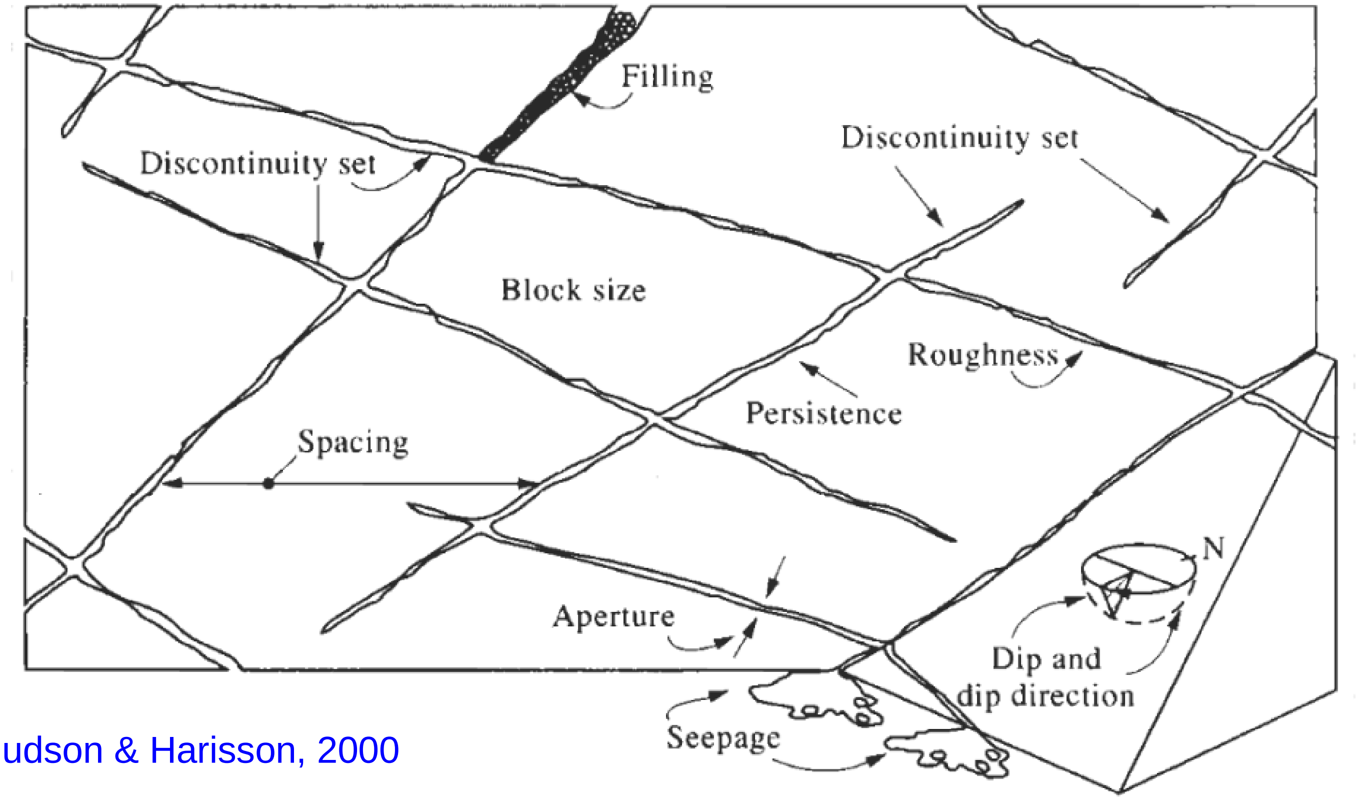
# Jointed rock mass

Jointed rock mass at Chalkidiki, Greece

# Micro-, meso- & macro-scale




Hudson & Harisson, 2000

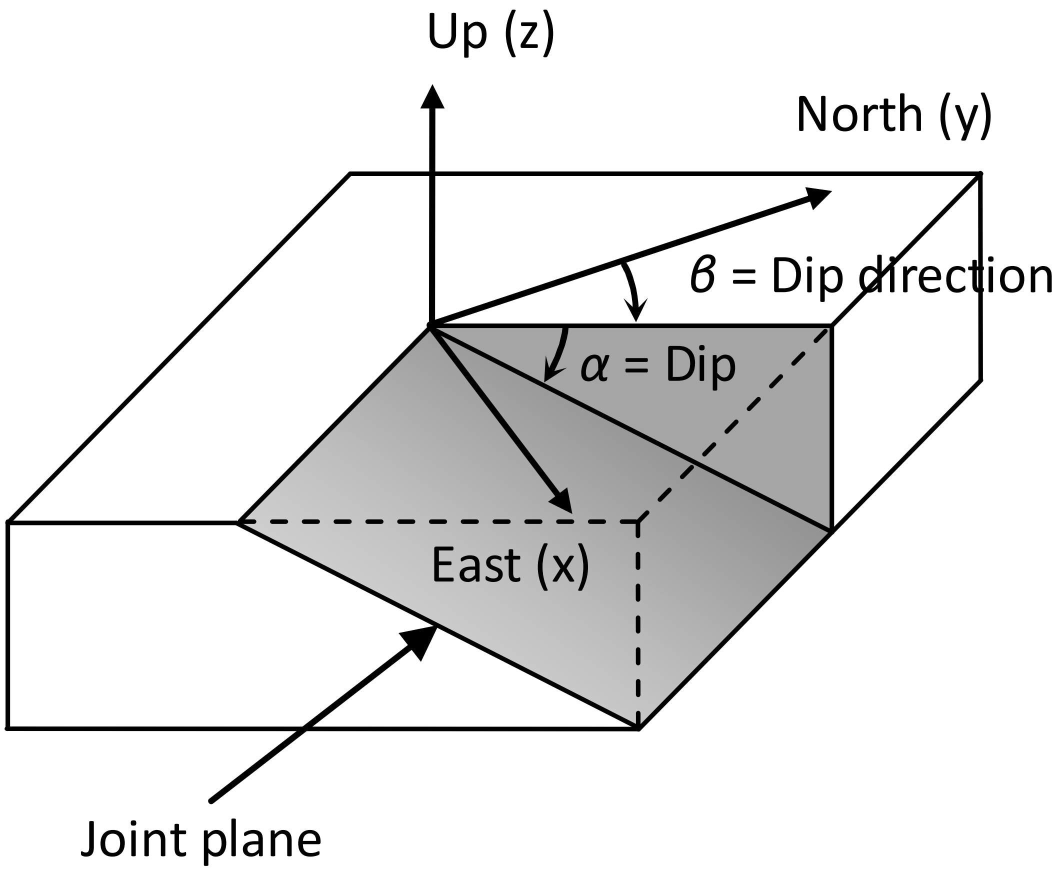
Microscale: the material of the rock blocks and the filling material
Mesoscale: the joints network
Macroscale: the rock mass

# Characterization of a discontinuity

Geometry: Dip angle α (positive downwards),

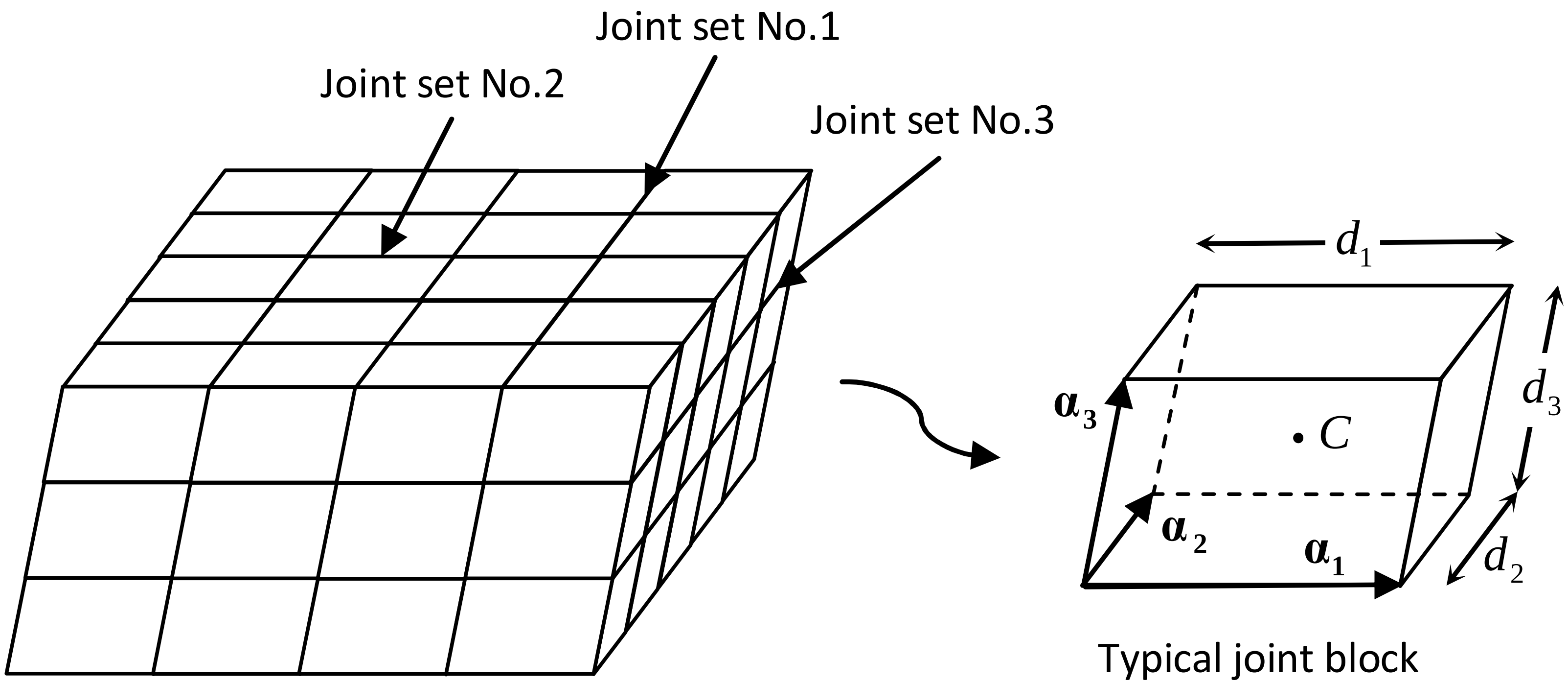
Dip direction β (positive clockwise from north)

Mechanical properties of the joint



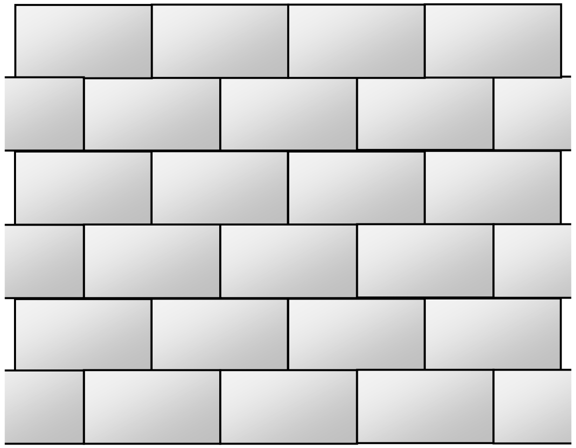
# 3D joint network

Uniform distribution of blocks and joints

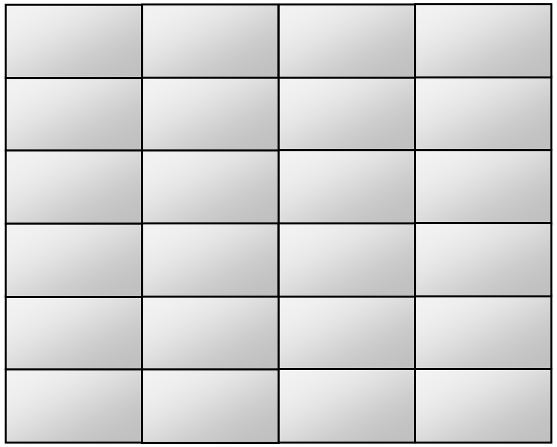



Three joint sets in 3D space forming parallelepipeds

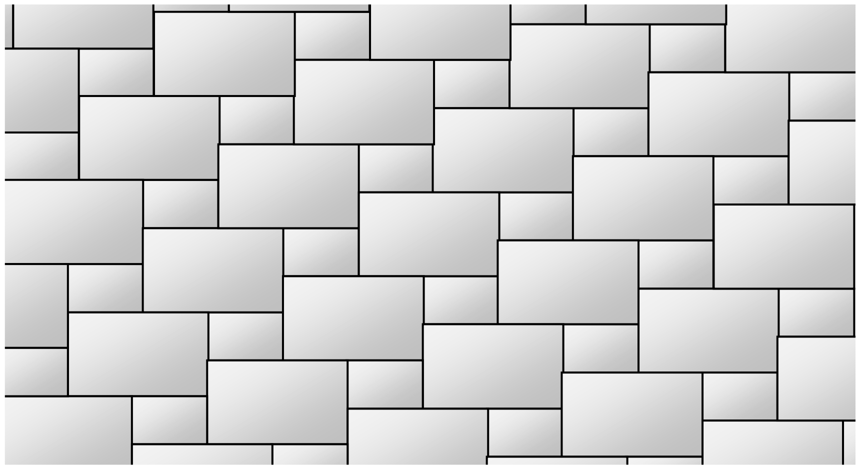
Extension to statistical distributions of the geometrical and mechanical properties is possible for pratical applications in rock engineering



Running bond



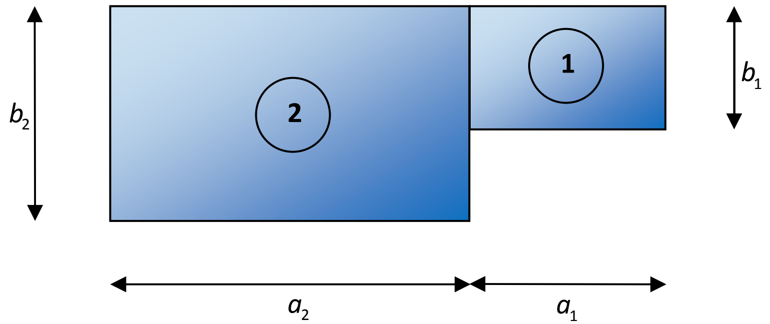
Stacked bond

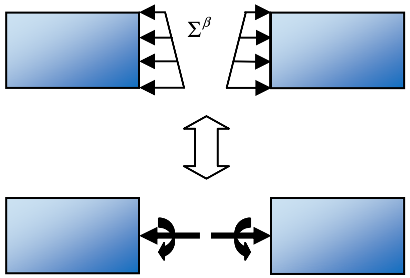


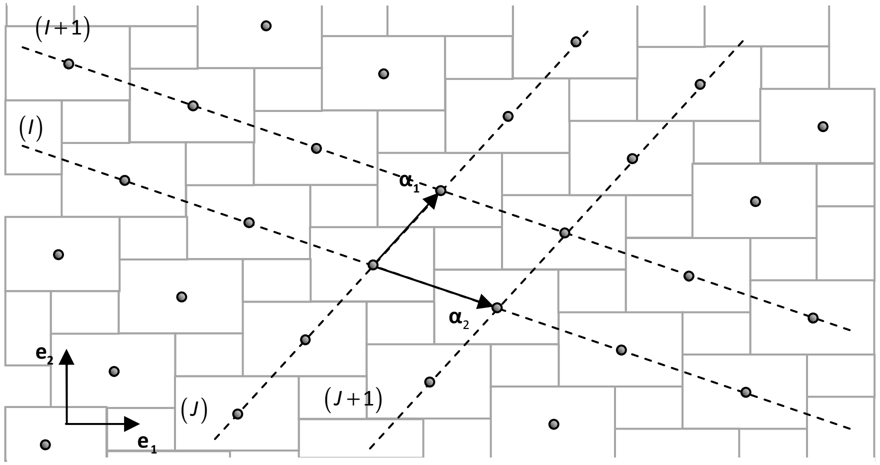
Interlocking
wall

Interlocking blocks at masonry wall in Peru (I.Vardoulakis 2006)

# Arrangement and contact of the building blocks



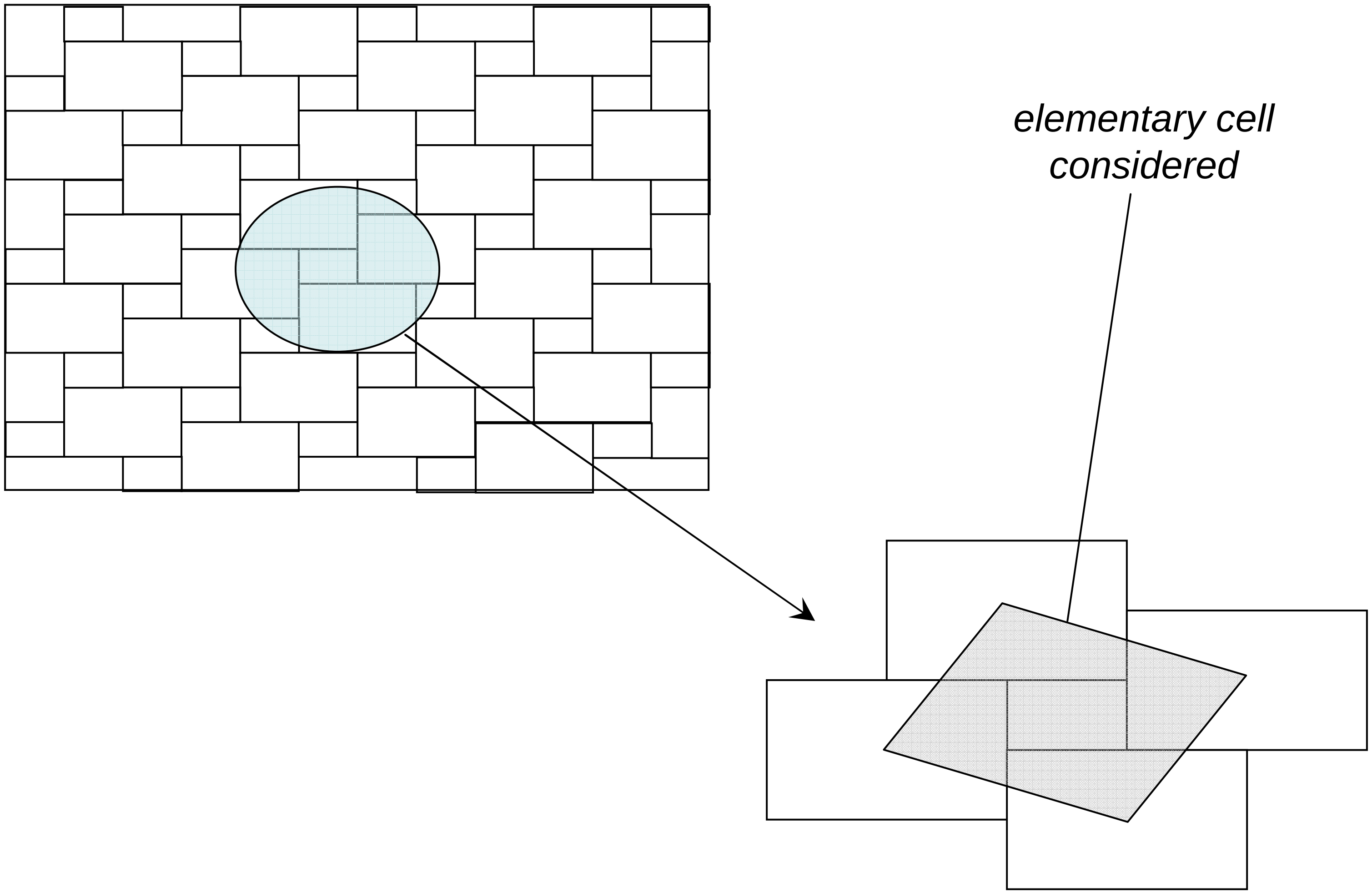




(G. Milani et al. 2006)

The building stones are considered rigid and the contact law linear elastic.

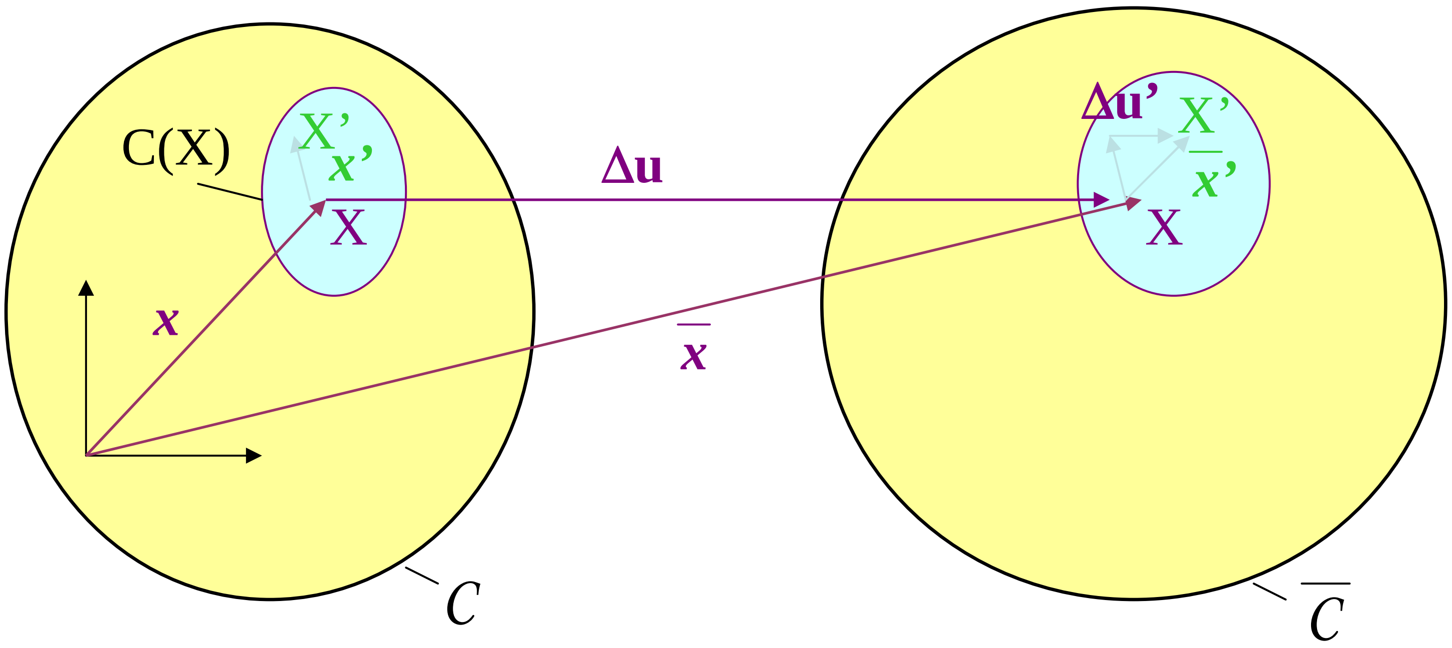
# bi-atomic pattern in a masonry structure




Stefanou, Sulem, Vardoulakis, *International Journal of Solids and Structures* 47 (2010) 1522–1536

Classical continuum : Assembly material points X

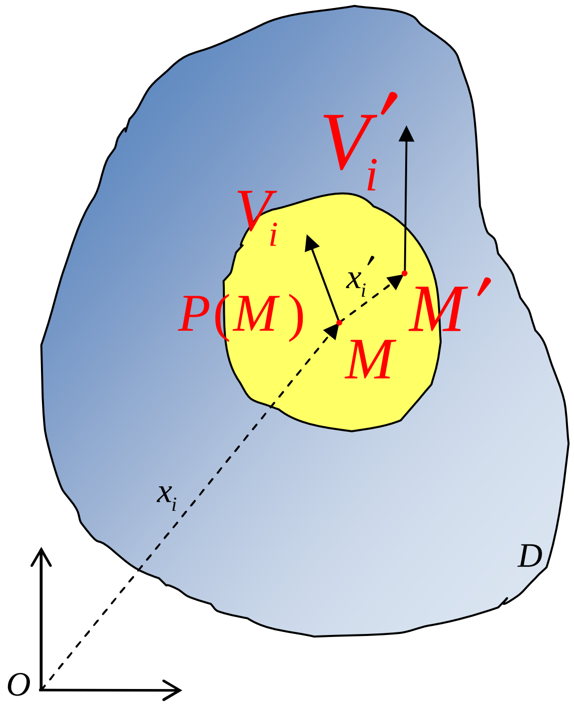
Generalised continuum : a micro-volume C(X) (continuum) is attached to each material points




Macro-déplacement : $\Delta \mathbf{u} = \bar{x} - x$

Micro-déplacement : $\Delta \mathbf{u}' = \bar{x}' - x'$

# Microstructure of generalized continua



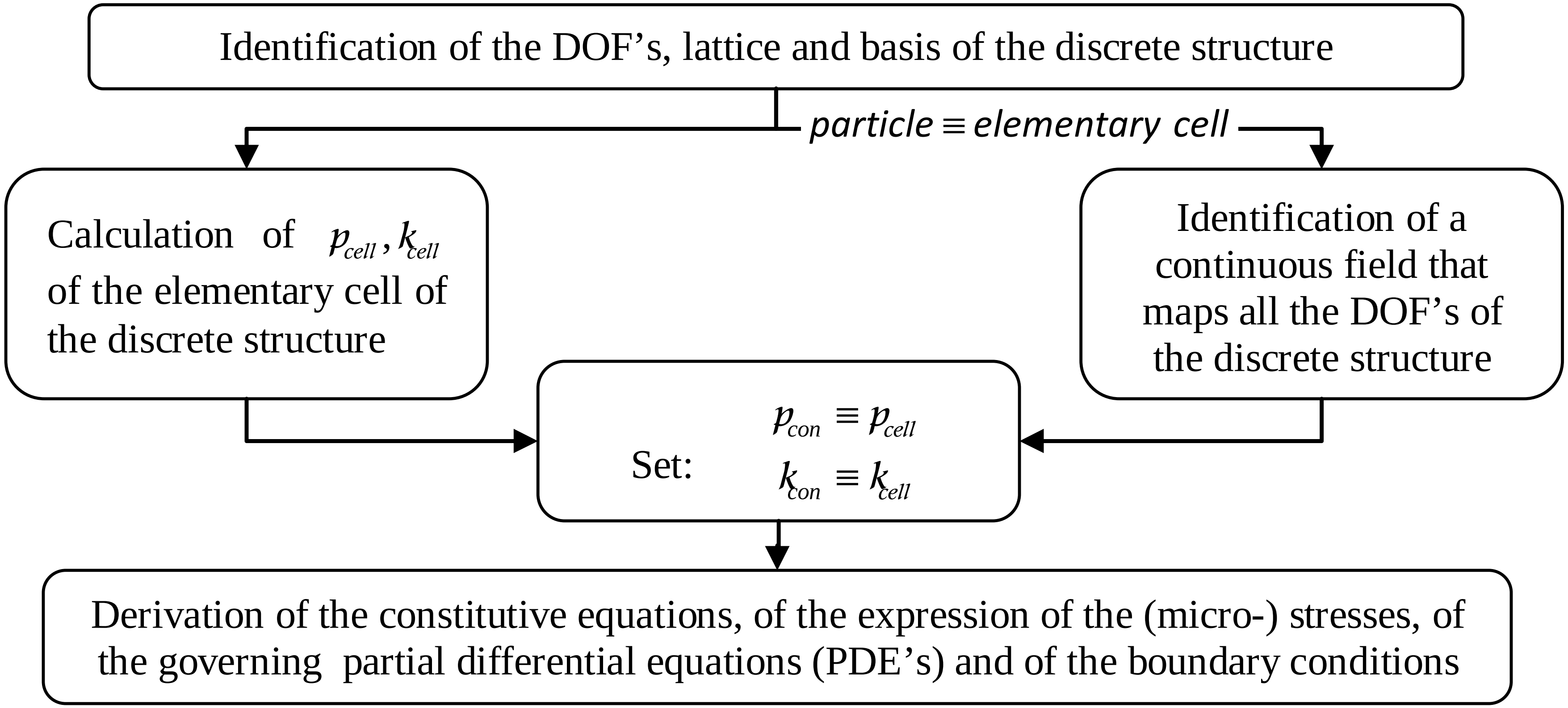

Taylor series expansion of the velocity field of the particle *P(M)*:

$$V_i' = V_i + \chi_{ij} x_j' + \chi_{ijk} x_j' x_k' + \chi_{ijk\ell} x_j' x_k' x_\ell' + \dots$$

$\chi_{\underbrace{ijk\dots\ell}_{\nu}}$ $\nu^{th}$ order microdeformation rate tensors

(P. Germain, 1973)

# Main steps


Identification of the DOF's, lattice and basis of the discrete structure
$particle \equiv elementary\ cell$
Calculation of $p_{cell}, k_{cell}$ of the elementary cell of the discrete structure
Identification of a continuous field that maps all the DOF's of the discrete structure
Set: $p_{con} \equiv p_{cell}$ $k_{con} \equiv k_{cell}$
Derivation of the constitutive equations, of the expression of the (micro-) stresses, of the governing partial differential equations (PDE's) and of the boundary conditions

# Generalized Differential Expansion Homogenization Technique

The target is the **derivation** of a continuum that has:

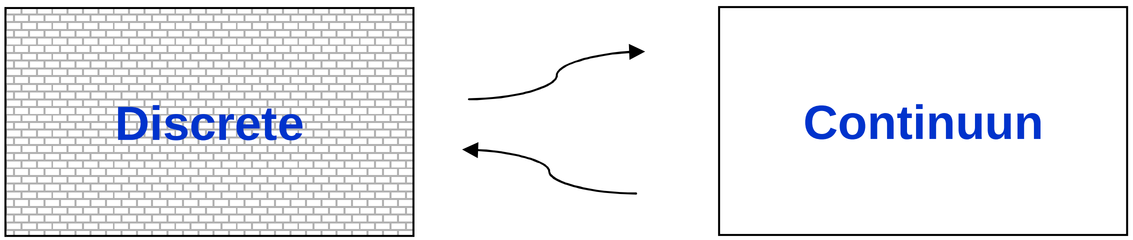
1. The same **kinematics** (DOFs, translational, rotational);

2. The same **elastic energy and kinetic energy** as the discrete system.

The method can:

3. Be applied **independently** of the elementary cell.

4. Avoids the **ad-hoc omission** of terms and averaging.




The derived continuum is not assumed a priori

# Kinematics

$$\left.\begin{aligned}
\dot{U}_i^{(I,J,K)} &\triangleq V_i'\left(\mathbf{r}'^{b^{(I,J,K)}}\right) \\
\dot{\Omega}_k^{(I,J,K)} &\triangleq -\frac{1}{2}\varepsilon_{ijk} V_{i,j}'\left(\mathbf{r}'^{b^{(I,J,K)}}\right) \\
\dot{\mathrm{E}}_{ij}^{(I,J)} &\triangleq \frac{1}{2}\left[V_{i,j}'\left(\mathbf{r}'^{b^{(I,J,K)}}\right) + V_{j,i}'\left(\mathbf{r}'^{b^{(I,J,K)}}\right)\right] = 0
\end{aligned}\right\} \leftrightarrow
\begin{aligned}
&\dot{U}_i^{(I,J,K)} = V_i^C \\
&\dot{\Omega}_k^{(I,J,K)} = -\frac{1}{2} e_{ijk}\chi_{ij} \\
&\chi_{11} = \chi_{22} = \chi_{33} = 0 \\
&\chi_{ij} = -\chi_{ji}
\end{aligned}$$

Antisymmetry of the first order microdeformation rate tensor $\chi_{ij}$.

# Constitutive law

$$\tau_{ij} = \frac{\partial p_{con}}{\partial V_{i,j}^{C}}$$

$$s_{ij} = -\frac{\partial p_{con}}{\partial \chi_{ij}} \qquad \text{where:} \quad p_{cell} = \frac{1}{V}\sum_{\beta=1}^{3}\left( F_i^{(b^A,\, b^B),\beta} \Delta \dot{U}_i^{(b^A,\, b^B),\beta} + M_i^{(b^A,\, b^B),\beta} \Delta \dot{\Omega}_i^{(b^A,\, b^B),\beta} \right)$$

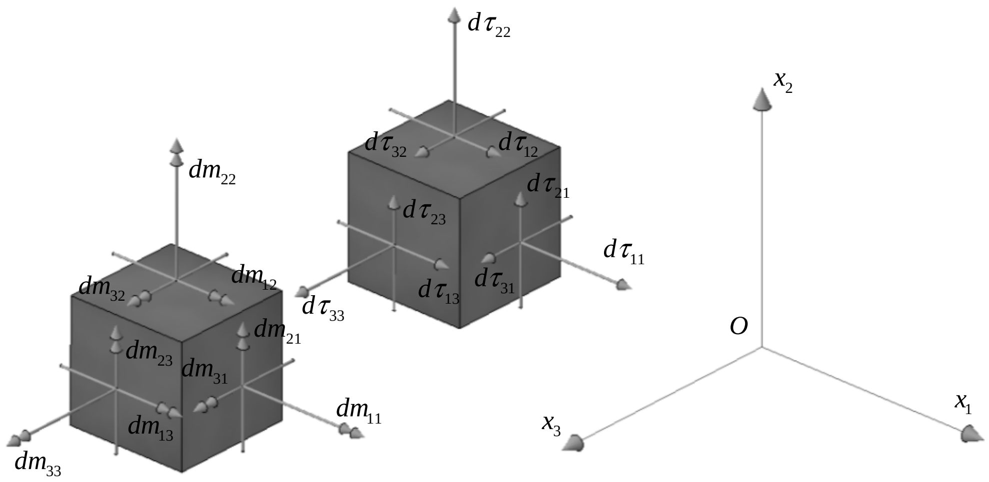
$$\nu_{ijk} = \frac{\partial p_{con}}{\partial \kappa_{ijk}}$$

# PDEs and Boundary Conditions

**3D anisotropic Cosserat** continuum:

$$\tau_{ij,j} + f_i = \rho \dot{V}_i^C$$

$$e_{kpq}\tau_{qp} + m_{k\ell,\ell} + \Psi_k = \frac{I_{kj}}{V_{P(M)}}\dot{\chi}_j$$



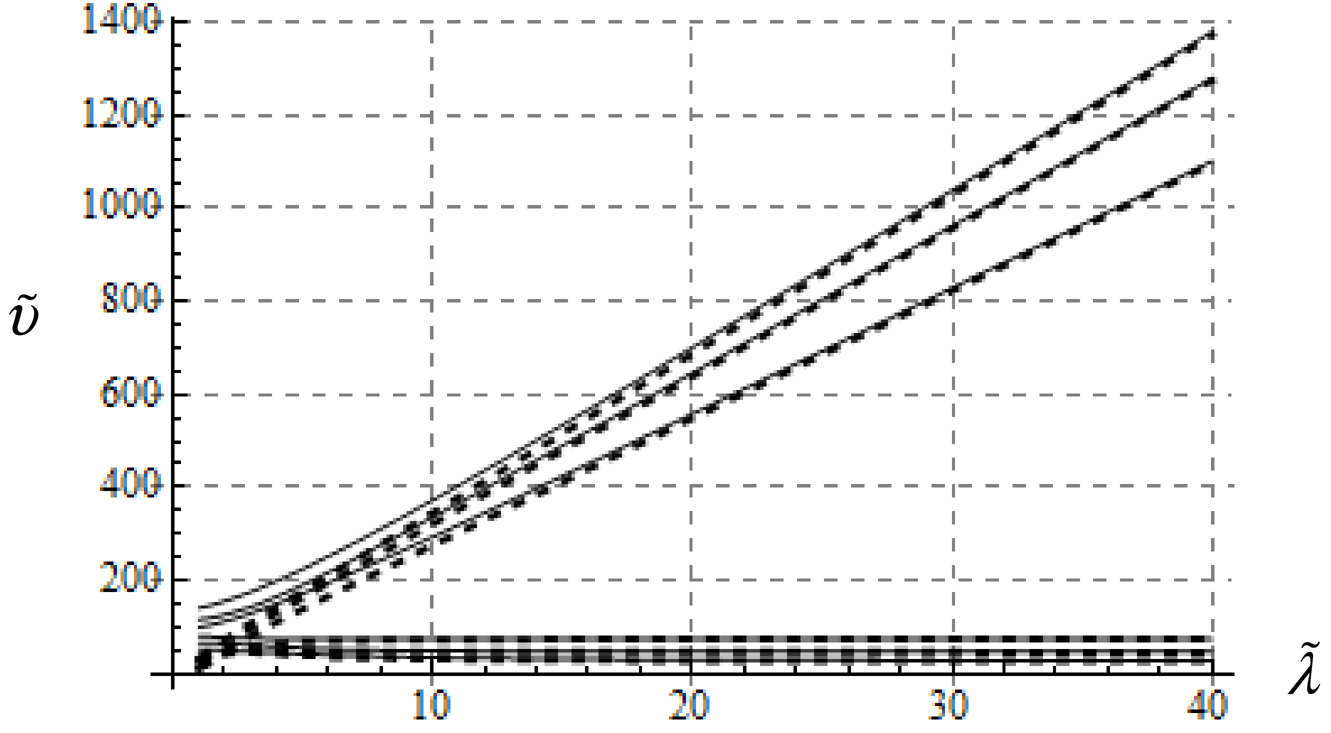

I.Stefanou, J.Sulem and I.Vardoulakis, Three-dimensional Cosserat homogenization of masonry structures: elasticity. Acta Geotechnica, 3, 1 (2008), 71-83.

External forces and generalized tractions (BCs):

$$T_i = \tau_{ij} n_j = F_i^{ex}$$

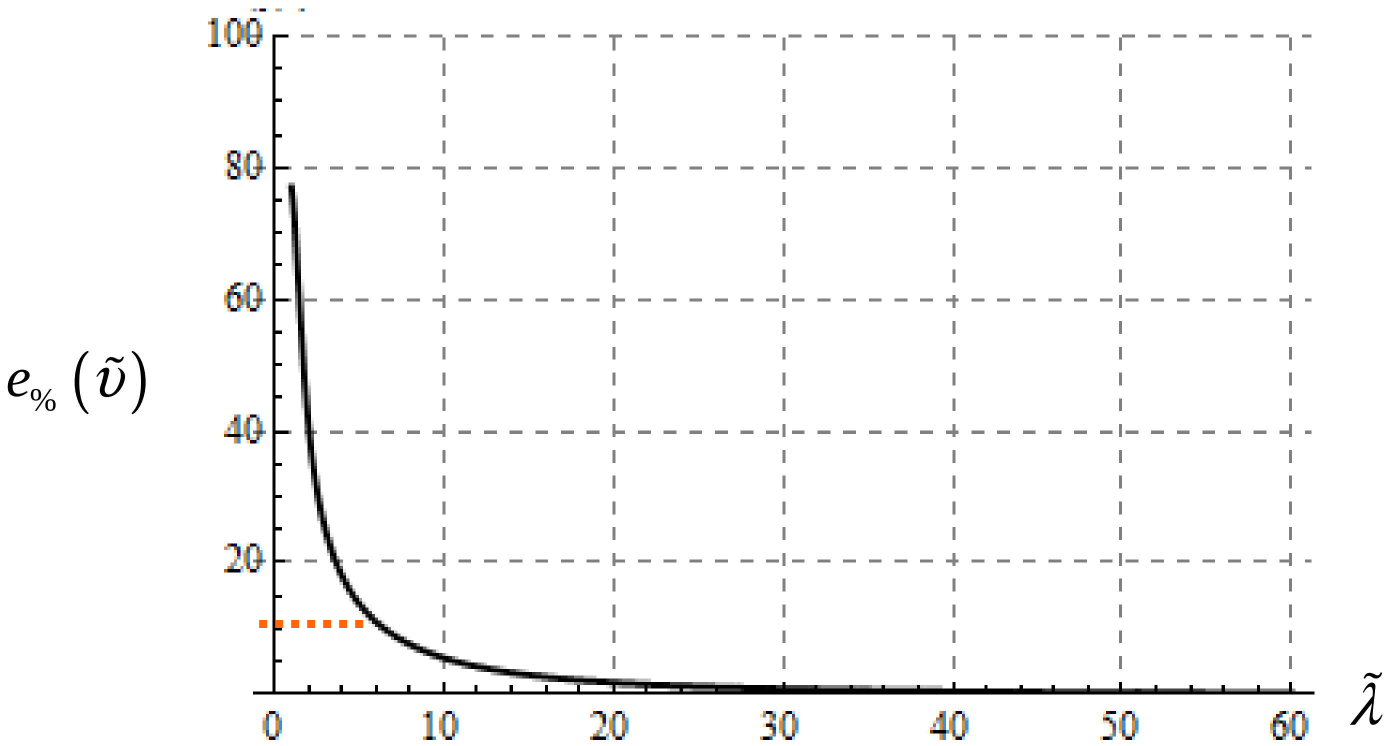
$$M_{ij} = \nu_{ijk} n_k = -\varepsilon_{ijk} M_k^{ex}$$

# Validation – Dispersion Curves

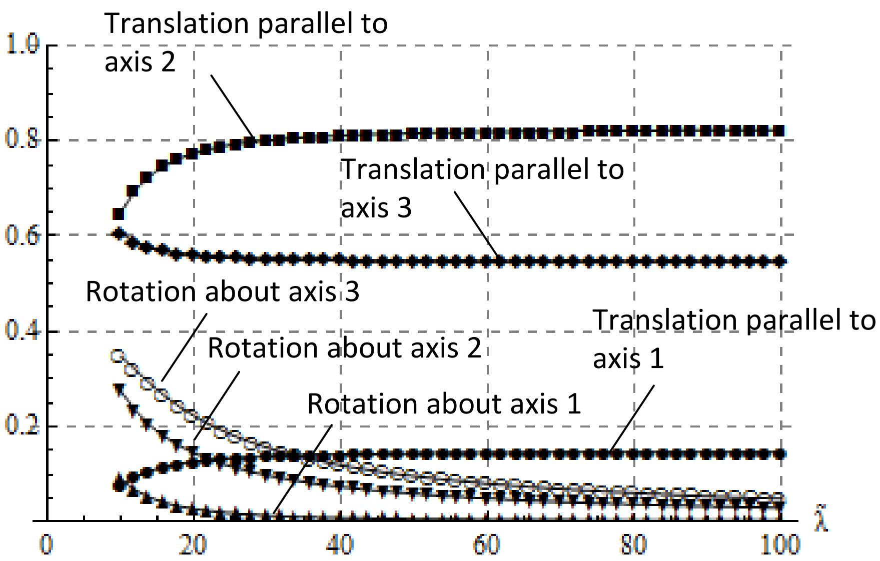



Six dispersion curves.
One for each degree of freedom.

# Validation – Relative error

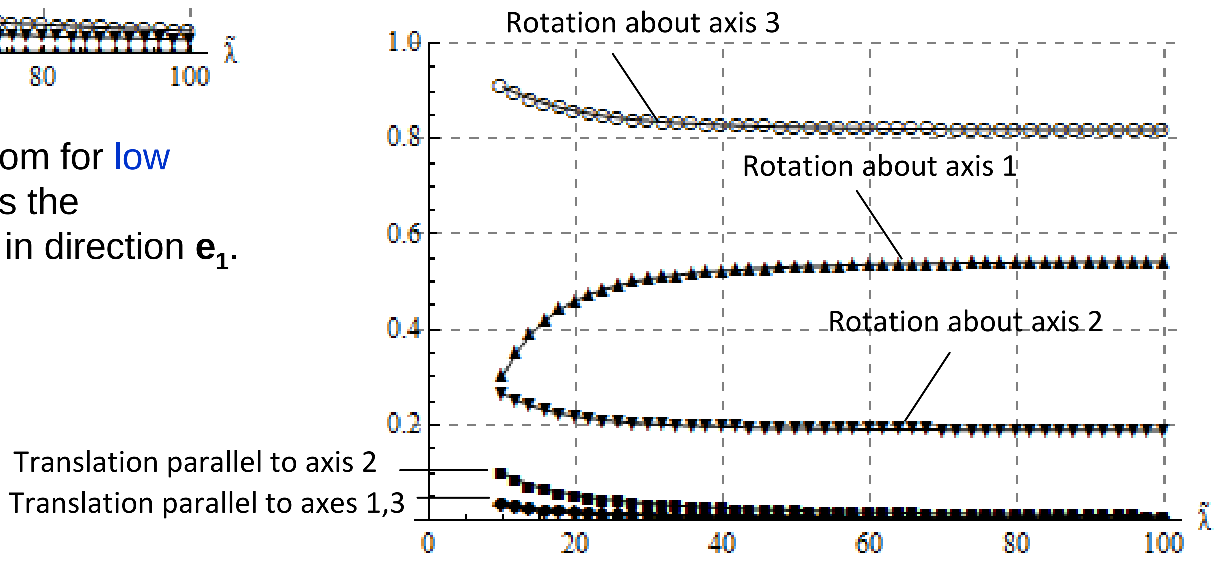


# Oscillation modes

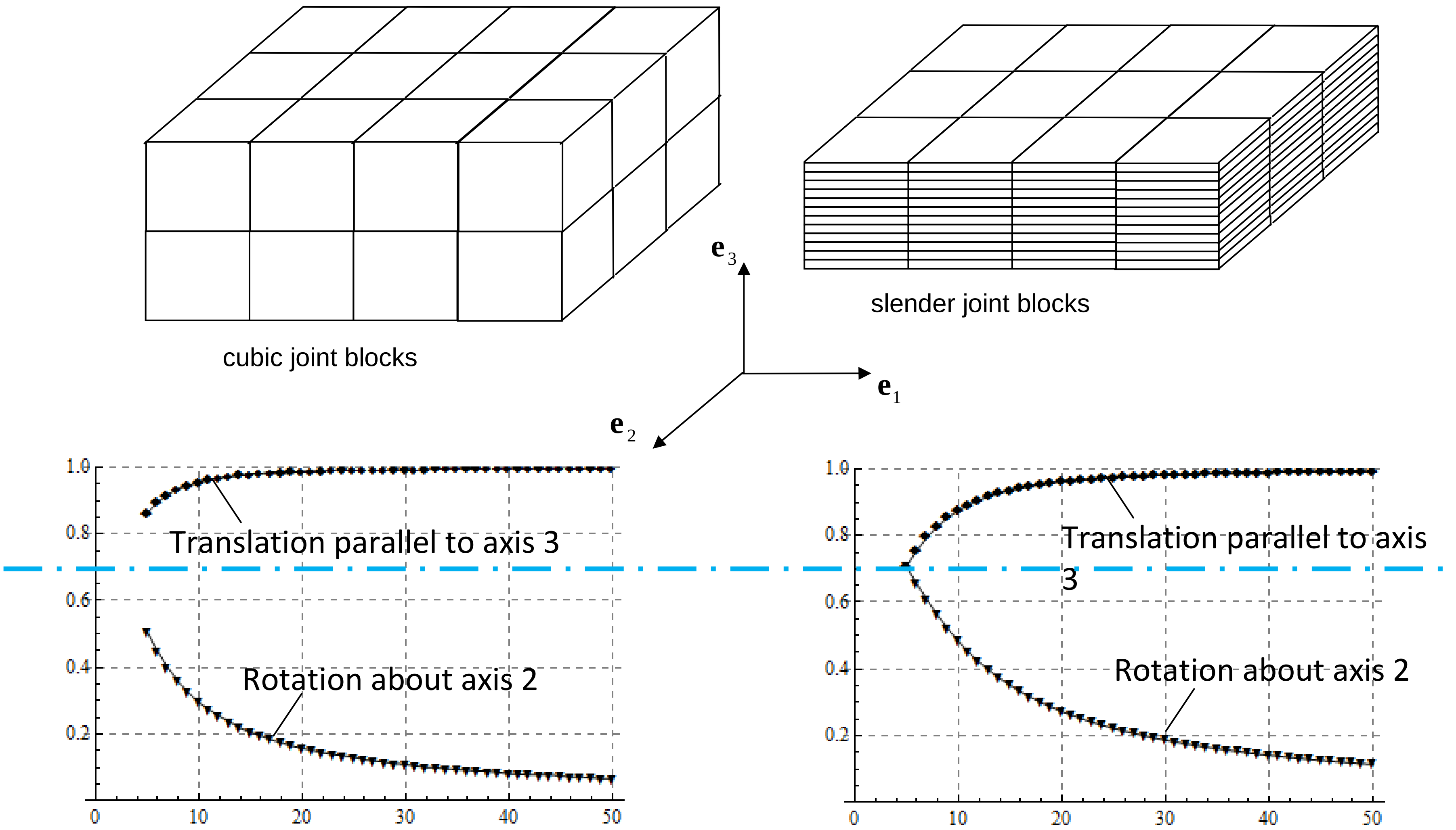



Amplitudes of the degrees of freedom for low frequencies (acoustic mode) versus the wavelength for propagating waves in direction $\mathbf{e_1}$.

Amplitudes of the degrees of freedom for high frequencies (optic mode) versus the wavelength for propagating waves in direction $\mathbf{e_1}$.



# Effect of block slenderness



# Conclusions

• Derivation of a 3D Cosserat continuum for discontinuous rock masses

• The continuum description is acceptable for wave length five times bigger than the block size

• Even for low frequencies the contribution of the Cosserat rotations is significant and increases for slender blocks

•Importance of Cosserat terms for dynamic applications