LIQUEFACTION

A Model of Liquefaction of Granular Materials in Isotropic Compression

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Dedicated to the memory of Prof. Ioannis Vardoulakis

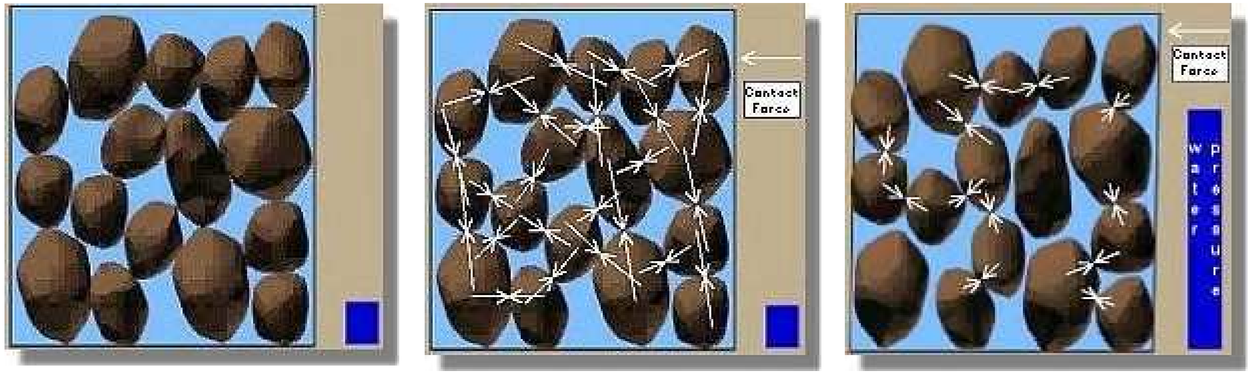
BACKGROUND -Definition of Soil Liquefaction

Liquefaction refers to the loss of strength in saturated, cohesionless soils due to the built-up of pore water pressures during loading.

Liquefaction is a phenomenon wherein a mass of soil loses a large percentage of its shear strength, when subjected to monotonic, cyclic or shock loading and flows in a manner resembling a liquid until the shear stress acting on the mass are as low as the reduced shear strength. [Sladen et al., 1985]

In a more general manner, soil liquefaction has been defined as the transformation *"from a solid state to a liquefied state as a consequence of increased pore pressure and reduced effective stress"* [Definition of terms…, 1978]

Some ground failures attributed to soil liquefaction are more correctly ascribed to "cyclic mobility" which results in limited soil deformation without liquid like flow.



Saturated soils consist of a solid skeleton of pores filled with water. The overall behavior of such a mixture depends on the effective stresses in the soil skeleton. If they are compressive and do not exceed failure criteria, the soil skeleton is able to support additional loads and behaves macroscopically like a solid body.

Under certain conditions, such as, for example, cyclic or shock loadings, the pore **water pressure** may **increase**, **reducing the effective stresses** and, consequently, **reducing the shearing resistance of saturated soil**. Under extreme conditions, the shearing strength may reach its residual value and saturated soil is said to have liquefied, as it behaves macroscopically as a liquid, which cannot support any load.

(The liquefaction behavior of saturated soils strongly depends on their initial state and structure. For example, it is easier to liquefy loose sand than dense sands.)

Practical Implications of Liquefaction

Niigata (Japan,1964)
earthquake destroyed all kinds of modern infra-structure due to soil liquefaction

When liquefaction occurs, the strength of the soil decreases and, the ability of a soil deposit to support foundations for buildings and bridges is reduced as seen in the photo of the overturned apartment complex buildings in Niigata*. Despite the extreme tilting, the buildings themselves suffered remarkably little structural damage.*

Sand boils and ground fissures were observed at various sites in Niigata.

Kobe (Japan, 1995)

Destructive earthquake, extensive liquefaction caused damage to geotechnical structures

Collapse of Nishinomiya bridge

(foundation deformations that are attributed to the effects of liquefaction. Ground cracks behind the quay walls and parallel to the water edge are indicative of the lateral ground movements that occurred. Sand boils are visible on the ground surface)

Soil liquefaction can damage structures in many ways as the supporting ground sinks or even pulls apart:

- Sand boils *which usually result in subsidence*
- Flow failures of slopes *involving very large down-slope movements of soil mass*
- Lateral spreads *resulting from the lateral displacements of gently sloping ground*
- Ground oscillations
- Loss of bearing capacity *and foundation failures,*
- Failure of retaining walls *due to increased lateral loads from liquefied backfill or loss of support from liquefied foundation soils*
- *Ground settlement, …*

Soil liquefaction is a multiscale, multiphysics problem, originating at the pore scale (microscale) level but rapidly propagating to the particle cluster scale (mesoscale) and finally to the specimen/structure scale (macroscale).

Identification of factors influencing liquefaction

Liquefaction is most often known as the result of a major earthquake, but can also be caused by construction practices such as blasting, vibroflotation and dynamic compaction. Generally, liquefaction will occur in loose clean to silty sands that are below the groundwater table but still close to the surface of the deposit. Historically, these sands are young Holocene age geomaterials. Below depths of approximately 20m, we generally do not see any evidence of liquefaction. This is due to the fact that deeper soils become compacted by the weight of the soil layer above.

- *Fabric and characteristics of soil grains (distribution of sizes, shape, composition, etc)*
- *Density of the soil*
- *Degree of saturation*
- *Permeability*
- *Type of loading*
- *Path of loading*
- *Undrained conditions*

Modeling liquefaction – related phenomena

The models of liquefaction-related phenomena can be roughly divided into three main groups, namely:

•Empirical approaches (experimental works) *see the book by Ishihara, 1996 and Youd & Idriss, 2001 Summary report, J.Geotech.Geoenviron.Eng., 127(4).*

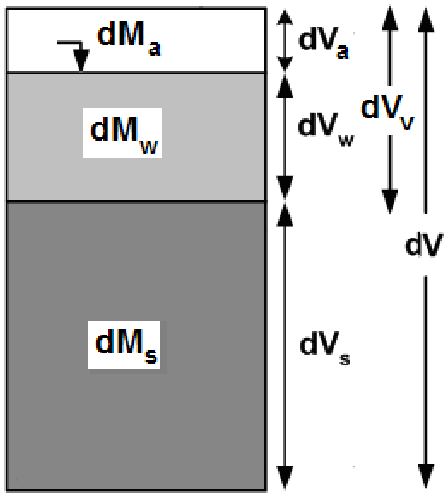
•Generalized or extended classical models (Biot-type approaches of poroelasticity theory,…)
Biot (1956) was the first to develop a 3D theory of wave propagation in fluid saturated poroelastic media. The eqns governing the interaction of the solid and fluid media have first been established by Biot for both quasi static and dynamic phenomena. The classical Biot theory is based on the assumption of linear elastic relation between the effective stresses and strains in the soil skeleton. An additional constitutive assumption is Darcy's law. These relations together with general principles of continuum mechanics led to the well known Biot consolidation eqns.

•Models containing some new ideas which are not found in classical descriptions (i.e. Bifurcation theory, Discrete elements, …).

Continuum mixtures theory of dynamical behaviour of water saturated media

Definitions

Partially saturated granular soils are considered to be mixtures of a solid phase (s), an aqueous phase (w) and air (a).



$$dV = dV_s + dV_v$$

$$dM = dM_s + dM_w + dM_a$$

- Porosity n and degree of saturation S

$$n = \frac{dV_v}{dV} = \frac{dV_w + dV_a}{dV}; \quad S = \frac{dV_w}{dV_v}$$

- The *bulk densities* of the constituents

$$\rho_s = \frac{dM_s}{dV_s}, \quad \rho_w = \frac{dM_w}{dV_w}, \quad \rho_a = \frac{dM_a}{dV_a}$$

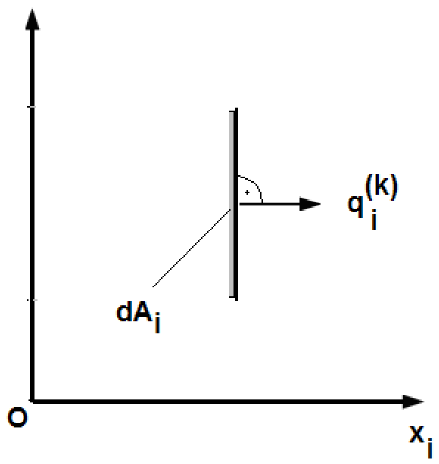
- The partial or relative densities

$$\rho^{(s)} = (1-n)\rho_s, \quad \rho^{(w)} = Sn\rho_w, \quad \rho^{(a)} = (1-S)n\rho_a$$

- Total density

$$\rho = (1-n)\rho_s + Sn\rho_w + (1-S)n\rho_a$$

Specific discharges



- Volume discharges $$q_i^{(k)} = \frac{dV_k}{dA_i dt}, \quad k = s, w, a$$
- Velocities $$v_i^{(k)} = \frac{dV_k}{dA_i^{(k)} dt}$$
- Mass discharges

$$m_i^{(k)} = \frac{dM_k}{dA_i dt} = \frac{dM_k}{dV_k}\frac{dV_k}{dA_i dt} = \rho_k q_i^{(k)}, \quad \rho_k = [\rho_s, \rho_w, \rho_\alpha]$$

For a statistically isotropic porous medium, for all cross sections the ratios of material surface areas are the same:

$$v_i^{(s)} = \frac{dV_s}{dA_i^{(s)} dt} = \frac{dV_s}{(1-n) dA_i\, dt} \Rightarrow$$

$$v_i^{(w)} = \frac{dV_w}{dA_i^{(w)} dt} = \frac{dV_w}{S n dA_i\, dt} \Rightarrow$$

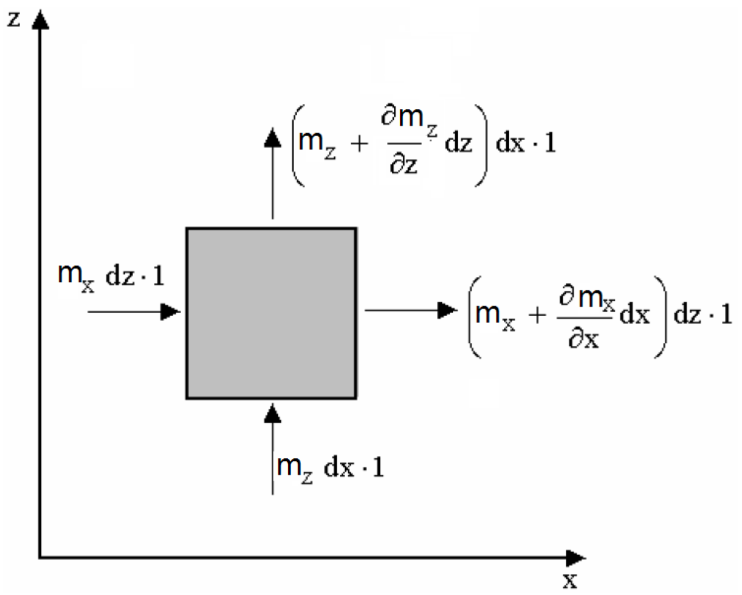
$$v_i^{(\alpha)} = \frac{dV_\alpha}{dA_i^{(\alpha)} dt} = \frac{dV_s}{(1-n) dA_i\, dt} \Rightarrow$$

$$q_i^{(s)} = (1-n) v_i^{(s)} = \frac{m^{(s)}}{\rho_s}$$

$$q_i^{(w)} = S n v_i^{(w)} = \frac{m^{(w)}}{\rho_w}$$

$$q_i^{(a)} = (1-S) n v_i^{(a)} = \frac{m^{(a)}}{\rho_a}$$

Conservation of mass



The basic principle which the specific mass discharges should obey is the conservation of mass and this principle should hold for every component of the material. In the case there is no mass source or sink, and there is diffusion between the water and air only, then the mass inside the volume (dxdz) equals the mass inflow minus the mass outflow

- Mass inflow $m_x dzdt + m_z dxdt$
- Mass outflow $(m_x + m_{x,x}dx)dzdt + (m_z + m_{z,z}dx)dxdt$
  - Mass balance=inflow - outflow

$$dm = -m_{x,x}dxdzdt - m_{z,z}dxdzdt$$

$$\frac{\partial \rho^{(k)}}{\partial t} + m_{i,i}^{(k)} = f^{(k)}, \quad k = s, w, a,$$
$$f^{(s)} = 0, \quad f^{(w)} = -f^{(a)} = f$$

$f^{(k)}$: *mass diffusion rate*

Conservation of mass

$$\frac{\partial \rho^{(k)}}{\partial t} + m_{i,i}^{(k)} = f^{(k)}, \quad k = s, w, a,$$
$$f^{(s)} = 0, \quad f^{(w)} = -f^{(a)} = f$$

Denote:

$$m_i^{(k)} = \rho_k q_i^{(k)} = \rho^{(k)} v_i^{(k)}; \quad \rho_k = [\rho_s, \rho_w, \rho_\alpha],$$
$$\rho^{(s)} = (1-n)\rho_s$$
$$\rho^{(w)} = Sn\rho_w$$
$$\rho^{(a)} = (1-S)n\rho_\alpha$$

$$\frac{\partial}{\partial t}[(1-n)\rho_s] + \frac{\partial}{\partial x_i}\left(\rho_s q_i^{(s)}\right) = 0$$
$$\frac{\partial}{\partial t}[Sn\rho_w] + \frac{\partial}{\partial x_i}\left(\rho_w q_i^{(w)}\right) = f$$
$$\frac{\partial}{\partial t}[(1-S)n\rho_\alpha] + \frac{\partial}{\partial x_i}\left(\rho_a q_i^{(\alpha)}\right) = -f$$

Partial stresses

The total stress of the medium is the sum of the partial stresses assigned to the three phases

$$\sigma_{ij} = \sigma_{ij}{}^{(s)} + \sigma_{ij}{}^{(w)} + \sigma_{ij}{}^{(a)}$$

The partial solid stress is called the intergranular stress since it is a measure of the intergranular forces transmitted among the grains at isolated contact points

$$\sigma_{ij}^{(s)} = \sigma'_{ij}$$

The partial stresses for the fluid and the gas are assumed to be pressures of the form

$$\sigma_{ij}^{(w)} = Sp^{w}\delta_{ij}$$

$$\sigma_{ij}^{(a)} = (1 - S)p^{a}\delta_{ij}$$

Displacements and strains

displacements of the solid, water and of the air at a point: $u_i^{(s)}, u_i^{(w)}, u_i^{(a)}$

the infinitesimal strain of the phases:

$$\varepsilon_{ij}^{(s)} = \frac{1}{2}\left(u_{i,j}^{(s)} + u_{j,i}^{(s)}\right); \quad \varepsilon_{ij}^{(w)} = u_{k,k}^{(w)}, \quad \varepsilon_{ij}^{(a)} = u_{k,k}^{(a)}$$

Solid – fluid interaction and Inertial effects

The total forces per unit volume acting on the solid, the fluid and the air: $F_{ti}^{(s)}, F_{ti}^{(w)}, F_{ti}^{(a)}$

These total forces are the sum of inertial and dissipative forces

*Following Biot (1956) the **inertial effects** are described by the eqns*

$$F_i^{(s)} = \frac{\partial^2}{\partial t^2}\left(\rho_{11}u_i^{(s)} + \rho_{12}u_i^{(w)} + \rho_{13}u_i^{(a)}\right),$$

$$F_i^{(w)} = \frac{\partial^2}{\partial t^2}\left(\rho_{12}u_i^{(s)} + \rho_{22}u_i^{(w)} + \rho_{23}u_i^{(a)}\right),$$

$$F_i^{(a)} = \frac{\partial^2}{\partial t^2}\left(\rho_{13}u_i^{(s)} + \rho_{23}u_i^{(w)} + \rho_{33}u_i^{(a)}\right)$$

The density coefficients represent the mass coupling parameters between the 3 constituents that take into account that the relative flow through the pores is not uniform. Their physical significance is nicely explained by Biot :

*Identification of coupling density coefficients**:*

(a) Assume no relative motion between solid – water - air $u_i^{(s)} = u_i^{(w)} = u_i^{(a)}$

$$F_i^{(s)} = \frac{\partial^2}{\partial t^2}(\rho_{11} + \rho_{12} + \rho_{13}) u_i^{(s)}, \qquad \rho^{(s)} = \rho_{11} + \rho_{12} + \rho_{13}$$ *Mass of solid per unit volume*

$$F_i^{(w)} = \frac{\partial^2}{\partial t^2}(\rho_{12} + \rho_{22} + \rho_{23}) u_i^{(w)}, \qquad \rho^{(w)} = \rho_{12} + \rho_{22} + \rho_{23}$$ *Mass of water per unit volume*

$$F_i^{(a)} = \frac{\partial^2}{\partial t^2}(\rho_{13} + \rho_{23} + \rho_{33}) u_i^{(a)} \qquad \rho^{(a)} = \rho_{13} + \rho_{23} + \rho_{33}$$ *Mass of air per unit volume*

(b) Assume water and air at rest and solid accelerating $u_i^{(w)} = u_i^{(a)} = 0$

$$F_i^{(s)} = \frac{\partial^2 \rho_{11} u_i^{(s)}}{\partial t^2},$$

$$F_i^{(w)} = \frac{\partial^2 \rho_{12} u_i^{(s)}}{\partial t^2},$$

$$F_i^{(a)} = \frac{\partial^2 \rho_{13} u_i^{(s)}}{\partial t^2}$$

The effect of forces exerted on the water and air by the accelerating solid to prevent any displacement (of water and air) is measured by the coupling coefficients ρ_{12} and ρ_{13}

***More explanation of the nature and range of the values of these coupling terms: see Biot (1956) and textbook of Vardoulakis and Sulem*

Dissipation effects

The seepage volume forces that are dissipative in nature and express the exchange of momentum between the three components, are given by the relationships :

Their nature depends on the fluid and air flow characteristics

$$F_{1i}^{(s)} = -F_{1i}^{(w)},\ F_{1i}^{(a)} = 0,$$
$$F_{2i}^{(s)} = -F_{2i}^{(a)},\quad F_{2i}^{(w)} = 0,$$
$$F_{3i}^{(s)} = 0,\quad F_{3i}^{(w)} = -F_{3i}^{(a)}$$

Combined inertia and dissipation effects

After Biot (1956), both inertia and dissipation effects are superimposed, so finally, the total forces per unit volume acting on each constituent take the following forms

$$F_{ti}^{(s)} = \frac{\partial^2}{\partial t^2}\left(\rho_{11}u_i^{(s)} + \rho_{12}u_i^{(w)} + \rho_{13}u_i^{(a)}\right) + F_{1i}^{(s)} + F_{2i}^{(s)} + F_{3i}^{(s)},$$
$$F_{ti}^{(w)} = \frac{\partial^2}{\partial t^2}\left(\rho_{12}u_i^{(s)} + \rho_{22}u_i^{(w)} + \rho_{23}u_i^{(a)}\right) + F_{1i}^{(w)} + F_{2i}^{(w)} + F_{3i}^{(w)},$$
$$F_{ti}^{(a)} = \frac{\partial^2}{\partial t^2}\left(\rho_{13}u_i^{(s)} + \rho_{23}u_i^{(w)} + \rho_{33}u_i^{(a)}\right) + F_{1i}^{(a)} + F_{2i}^{(a)} + F_{3i}^{(a)}$$

Balance of linear momentum

The eqns of balance of linear momentum for each constituent take the following forms

$$\frac{\partial \sigma'_{ij}}{\partial x_j} + f_i^{(s)} = F_{ti}^{(s)} = \frac{\partial^2}{\partial t^2}\left(\rho_{11}u_i^{(s)} + \rho_{12}u_i^{(w)} + \rho_{13}u_i^{(a)}\right) + F_{1i}^{(s)} + F_{2i}^{(s)} + F_{3i}^{(s)},$$

$$S\frac{\partial p_w}{\partial x_i} + f_i^{(w)} = F_{ti}^{(w)} = \frac{\partial^2}{\partial t^2}\left(\rho_{12}u_i^{(s)} + \rho_{22}u_i^{(w)} + \rho_{23}u_i^{(a)}\right) + F_{1i}^{(w)} + F_{2i}^{(w)} + F_{3i}^{(w)},$$

$$(1-S)\frac{\partial p_a}{\partial x_i} + f_i^{(a)} = F_{ti}^{(a)} = \frac{\partial^2}{\partial t^2}\left(\rho_{13}u_i^{(s)} + \rho_{23}u_i^{(w)} + \rho_{33}u_i^{(a)}\right) + F_{1i}^{(a)} + F_{2i}^{(a)} + F_{3i}^{(a)}$$

p_w, p_a *fluid and gas pressures*

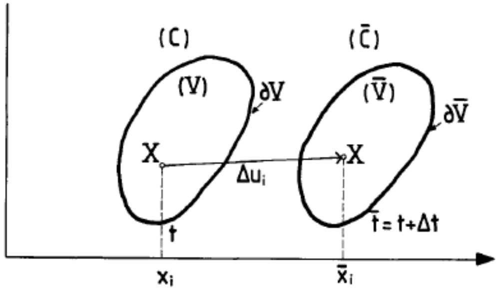
σ'_{ij} *effective stress acting on the solid skeleton*

$f_i^{(s)}$, $f_i^{(w)}$, $f_i^{(a)}$ *external body forces exerted on each component by various fields (i.e. gravitational field and the pressure field of the fluid and the air)*

Linearization of mass balance eqns

*Assume that the porous medium is initially at equilibrium and subsequently experiences a small transition (**infinitesimal increments**) in the various dependent variables*

$$(1-n)\frac{\partial \rho_s}{\partial t} - \rho_s \frac{\partial n}{\partial t} + (1-n)\rho_s v_{i,i}^{(s)} = 0$$



$$n \to n + \Delta n;\ \rho_s \to \rho_s + \Delta\rho_s;$$
$$v_i^{(s)} \to v_i^{(s)} + \Delta v_i^{(s)}$$

$$-\frac{\partial \Delta n}{\partial t} + \frac{1-n}{\rho_s}\frac{\partial \Delta\rho_s}{\partial t} + (1-n)\frac{\partial \Delta\varepsilon_{kk}}{\partial t} = 0$$

$$\rho_w \to \rho_w + \Delta\rho_w$$
$$S \to S + \Delta S$$

$$S\frac{\partial \Delta n}{\partial t} + n\frac{\partial \Delta S}{\partial t} + \frac{nS}{\rho_w}\frac{\partial \Delta\rho_w}{\partial t} + q_{i,i}^{(w)} + nS\frac{\partial \Delta\varepsilon_{kk}}{\partial t} = f \quad *$$

$$\rho_a \to \rho_a + \Delta\rho_a$$

$$(1-S)\frac{\partial \Delta n}{\partial t} - n\frac{\partial \Delta S}{\partial t} + \frac{(1-S)n}{\rho_a}\frac{\partial \Delta\rho_a}{\partial t} + q_{i,i}^{(a)} + n(1-S)\frac{\partial \Delta\varepsilon_{kk}}{\partial t} = -f \quad *$$

**** If we ignore diffusion between water and air: $f = 0$***

Linearization of balance of linear momentum eqns

Ignoring mass coupling parameters (ρ_{12}, ρ_{13}, ρ_{23}=0) we may derive the following linearized eqns of conservation of linear momentum

$$\frac{\partial \Delta \sigma'_{ij}}{\partial x_j} + f_i^{(s)} = (1-n)\rho_s \frac{\partial^2 u_i^{(s)}}{\partial t^2} + F_{1i}^{(s)} + F_{2i}^{(s)} + F_{3i}^{(s)},$$

$$S\frac{\partial \Delta p_w}{\partial x_j} + f_i^{(w)} = Sn\rho_w \frac{\partial^2 u_i^{(s)}}{\partial t^2} + \rho_w \frac{\partial Q_i^{(w)}}{\partial t} + F_{1i}^{(w)} + F_{2i}^{(w)} + F_{3i}^{(w)},$$

$$(1-S)\frac{\partial \Delta p_a}{\partial x_j} + f_i^{(a)} = (1-S)n\rho_a \frac{\partial^2 u_i^{(s)}}{\partial t^2} + \rho_a \frac{\partial Q_i^{(a)}}{\partial t} + F_{1i}^{(a)} + F_{2i}^{(a)} + +F_{3i}^{(a)}$$

$$Q_i^{(w)} = Sn\left(v_i^{(w)} - v_i^{(s)}\right)$$

$$Q_i^{(a)} = (1-S)n\left(v_i^{(a)} - v_i^{(s)}\right)$$

:the relative specific discharge of water and air

Constitutive eqns

Motivated from classical fluid mechanics we assume the following eqn for slightly compressible fluids

$$\Delta\rho_w = \rho_w \beta_w \Delta p_w$$

β_w : *expresses the compressibility of the fluid (barotropic flow, Malvern 1969)*

$$\Delta\rho_a = \frac{\rho_a}{p_a}\Delta p_a$$

Boyle's law of perfect gases

The above eqns mean that changes in fluids volume are only due to pressure changes. Similarly, we assume that changes in particle's volume are due to changes in the pore pressure and due to changes in the intergranular stress, i.e.

$$\Delta V_s = -\beta_s \Delta p_w V_s + \beta_p \Delta p' V_s; \quad \Delta p' = \Delta\sigma'_{kk}/3$$

$$\rho_s = \frac{M_s}{V_s} \Rightarrow \Delta\rho_s = -\frac{M_s}{V_s^2}\Delta V_s = -\rho_s \frac{-\beta_s \Delta p_w V_s + \beta_p \Delta p' V_s}{V_s} \Rightarrow \boxed{\Delta\rho_s = \rho_s\left(\beta_s \Delta p_w - \beta_p \Delta p'\right)}$$

β_s *:the compressibility of the solid particle material,* β_p *: the compressibility of the solid particle material due to concentrated forces at the contact points among particles*

Constitutive eqns

For the degree of saturation a constitutive law of the following form is assumed (Vardoulakis and Beskos, 1986)

$$\Delta S = -S\left(\alpha\,\Delta\varepsilon + \beta_S \Delta p_w\right)$$

The effect of increment of air pressure is ignored (valid for nearly saturated material, i.e. for 1-S<<1)

Furthermore, we assume that the seepage forces are proportional to the relative velocities according to Darcy's law

$$\begin{aligned}
F_{1i}^{(s)} &= b^{(w)}\left(v_i^{(s)} - v_i^{(w)}\right); & F_{1i}^{(w)} &= b^{(w)}\left(v_i^{(w)} - v_i^{(s)}\right),\\
F_{2i}^{(s)} &= b^{(a)}\left(v_i^{(s)} - v_i^{(a)}\right); & F_{2i}^{(a)} &= b^{(a)}\left(v_i^{(a)} - v_i^{(s)}\right),\\
F_{3i}^{(s)} &= 0; & F_{3i}^{(w)} &= -F_{3i}^{(a)} = 0
\end{aligned}$$

the proportionality constants are given as $b^{(w)} = \rho_w g / k_w$ $\quad b^{(a)} = \rho_a g / k_a$

k_w, k_a *are the coefficients of permeability of the skeleton for the fluid and air*

$$k_w = \frac{k\rho_w g}{v_w}$$

k *is Muskat's permeability [L²]*

v_w *is kinematic viscosity of the water [FL⁻²T]*

Soil behavior in isotropic compression

We will only touch upon liquefaction phenomena related to the action of cyclic isotropic compression, i.e. loading-unloading of granular material along the hydrostatic axis in the Haigh-Westergaard stress space. For this purpose we use the experimental results of Fragaszy and Voss (1984).

In these experimental tests on loose and dense sand, the initial dry density of the sand, the initial effective stress state, and the magnitude of the total stress cycle were systematically varied to determine the influence of each on liquefaction potential. Herein, the compression induced liquefaction is modeled within the frame of the theory for a three-phase elastoplastic porous medium, which provides a relatively simple theoretical basis for the observed phenomena.

Soil behavior in isotropic compression *Fully drained soil behavior*

In the first step one performs the calibration of finite stress-strain law from a single-cycle fully drained isotropic compression test. During the loading cycle the total confining pressure (σ) and the water pressure (p_w) are monitored. Starting from a stress-free configuration that is characterized by an initial porosity and an initial effective pressure an isotropic compression is performed resulting in a new configuration of the body

$$C_0 : n_0,\ p'_0 \qquad\qquad C_0 \to C : n,\ p'$$

• *the total volumetric strain obeys a power-law of the following form* $\varepsilon = \left(\dfrac{p'}{K}\right)^N$

• *for the first unloading branch after ultimate yielding configuration* C_Y *we assume the following finite elastic strain-stress phenomenological power-law*

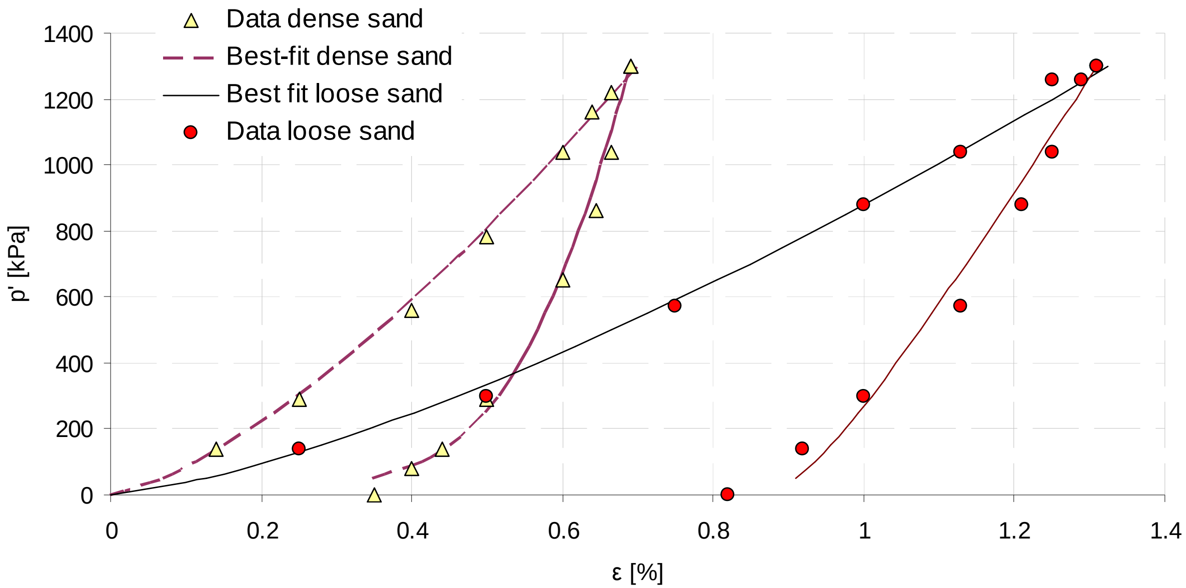
$$\varepsilon^e = A\left(\frac{p'_Y}{p'}\right)^M \left(\frac{p'_Y}{p'} - 1\right)$$

ε^e *:denotes the elastic strain derived by subtracting from the strain at yield the unloading strain*

The constant material parameters **(K, N, M, A)** *may be found by least squares regression analysis*

Soil behavior in isotropic compression *Fully drained soil behavior*

Experimental and best-fitted data referring to the loading-unloading behavior *of loose and dense Eniwetok beach sand (Fragaszy and Voss,1984).*



$$\varepsilon = \left(\frac{p'}{K}\right)^N$$

$$\varepsilon^e = A\left(\frac{p'_Y}{p'}\right)^M\left(\frac{p'_Y}{p'}-1\right)$$

or better

$$\varepsilon^e = \frac{p'_Y}{A}\left(1-\frac{p'_Y}{p'}\right)^M$$

Eniwetok sand	$\rho^{(s)}$ [g/cm^3]	n_0 [--]	K [kPa]	N [--]	A [--]	M [--]
Loose	1.53	0.456	524922	0.7205	0.00368170	0.9621
Dense	1.67	0.406	1410000	0.7098	0.00157643	0.7508

Soil behavior in isotropic compression *Fully drained soil behavior*

The incremental relations for the loading and unloading paths of the solid skeleton can be found by formal differentiation

- *Loading*

$$\varepsilon = \left(\frac{p'}{K}\right)^N$$

$$\Delta\varepsilon = \beta\Delta p'; \quad \beta = \frac{N}{K}\left(\frac{p'}{K}\right)^{N-1}$$

- *Unloading*

$$\varepsilon^e = A\left(\frac{p_Y'}{p'}\right)^M \left(\frac{p_Y'}{p'} - 1\right)$$

$$\Delta\varepsilon^e = -\beta_e \Delta p'; \quad \beta_e = \frac{A}{p_Y'^{\,M}}\left(M - (M-1)\frac{p_Y'}{p'}\right) p'^{\,M-1}$$

Soil behavior in isotropic compression *Undrained soil behavior*

For the simulation of a single loading cycle isotropic compression test in undrained conditions:

$$\left.\begin{aligned} &\Delta\rho_s = \rho_s\left(\beta_s \Delta p_w - \beta_p \Delta p'\right) \\ &-\frac{\partial \Delta n}{\partial t} + \frac{1-n}{\rho_s}\frac{\partial \Delta \rho_s}{\partial t} + (1-n)\frac{\partial \Delta \varepsilon_{kk}}{\partial t} = 0 \end{aligned}\right\}$$

$$\boxed{\frac{\Delta n}{1-n} = (1-a)\left(\beta_s \Delta p_w + \Delta\varepsilon\right); \quad \varepsilon = \varepsilon_{kk}} \qquad a = \frac{\beta_p}{\beta}$$

In which $\Delta p' = \frac{1}{\beta}\Delta\varepsilon$

$$\boxed{\beta = \begin{cases} \beta & loading \\ \beta_e & unloading \end{cases}}$$

denotes ***the bulk compressibility of the drained solid skeleton***

Assuming incompressible grains:
Volume changes are only due to changes in porosity

$$\boxed{\frac{\Delta n}{1-n} = \Delta\varepsilon}$$

Soil behavior in isotropic compression *Undrained soil behavior*

By introducing the eqns

$$\left.\begin{aligned}\Delta\rho_s &= \rho_s\left(\beta_s\Delta p_w - \beta_p\Delta p'\right)\\ \Delta\rho_w &= \rho_w\beta_w\Delta p_w\\ \Delta S &= S\frac{1-S}{p_w}\Delta p_w\end{aligned}\right\}$$

into the continuity for the fluid phase

$$S\frac{\partial\Delta n}{\partial t} + n\frac{\partial\Delta S}{\partial t} + \frac{nS}{\rho_w}\frac{\partial\Delta\rho_w}{\partial t} + q_{i,i}^{(w)} + nS\frac{\partial\Delta\varepsilon_{kk}}{\partial t} = 0$$

$q_{i,i}^{(w)} = 0$ undrained conditions

$$\Delta p_w = \frac{1}{1+\frac{1-n}{n}\cdot\frac{\beta_s}{\beta_w\left(1+\frac{1-S}{p_w\beta_w}\right)}}\cdot\frac{1}{n\beta_w\left(1+\frac{1-S}{p_w\beta_w}\right)}\cdot\Delta\varepsilon$$

Assuming incompressible grains:

$$\Delta p_w = \frac{1}{n\beta_w\left(1+\frac{1-S}{p_w\beta_w}\right)}\cdot\Delta\varepsilon$$

Soil behavior in isotropic compression *Undrained soil behavior*

Next, two tests on Eniwetok sand are simulated with the aid of the above continuum theory. In each simulation,

- *the parameters of the sand and water are given,*
- *the degree of saturation is assumed and*
- *the initial and peak confining total pressures and pore water pressure are initialized.*
- *The initial effective stress is found by recourse to the Terzaghi's effective stress principle*

Then, the increment of the bulk compressibility of the skeleton is calculated depending on loading or unloading.

$$\beta = \begin{cases} \beta & loading \\ \beta_e & unloading \end{cases}$$

$$\beta = \frac{N}{K}\left(\frac{p'}{K}\right)^{N-1}$$

$$\beta_e = \frac{A}{p_Y'^{\,M}}\left(M - (M-1)\frac{p_Y'}{p'}\right)p'^{\,M-1}$$

Soil behavior in isotropic compression *Undrained soil behavior*

The strain increment is next calculated from the increment of the total stress and with the aid of the following eqns and Terzaghi's effective stress principle and assuming incompressible grains

$$\Delta\varepsilon = \beta \Delta p'$$

$$\Delta p_w = \frac{1}{n\beta_w\left(1+\dfrac{1-S}{p_w\beta_w}\right)}\cdot\Delta\varepsilon$$

$$\Delta\varepsilon = \frac{1}{1/\beta + 1\Big/ n\beta_w\left(1+\dfrac{1-S}{p_w\beta_w}\right)}\cdot\Delta\sigma$$

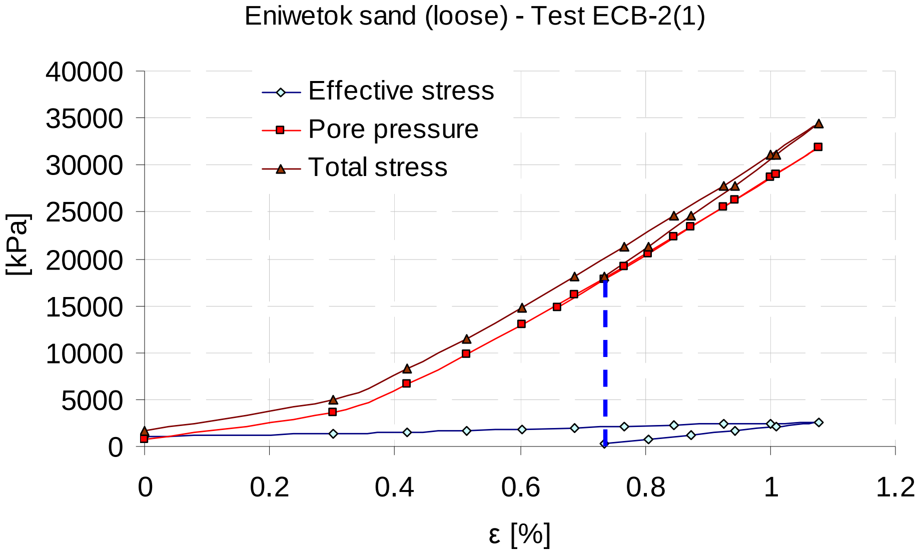
In a next stage all the dependent parameters are updated and the next simulation cycle begins.

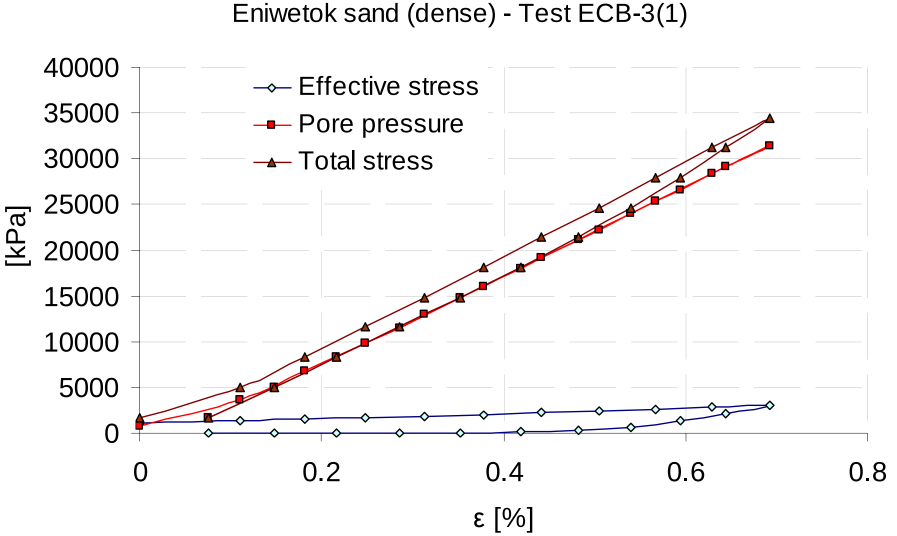
The best guess of the degree of saturation that is initially unknown, is found by minimization of the squared error of the peak pore water pressure recorded in the experiment and that predicted by the model.

With this algorithm the results of two simulations of a loose and dense Eniwetok sand with the same initial confining and pore pressures are displayed

Soil behavior in isotropic compression

Undrained soil behavior





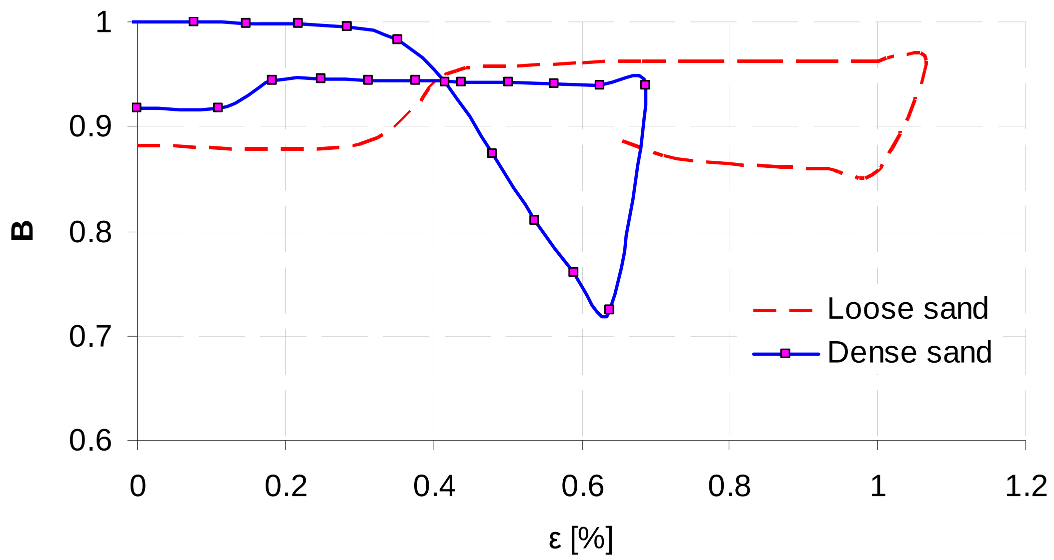
- *the loose sand liquefies before the end of the test*
- *dense sand exhibits positive effective stress until the end of the cycle*

- *The liquefaction of the loose sand is attributed to plastic pore volume decrease during unloading and contraction of sand.*
If this pore volume reduction is significant, then it leads to considerable pore pressure increase that causes liquefaction of sand.

- *These results are in accordance with the experimental test results (Fragaszy and Voss, 1984).*

Soil behavior in isotropic compression *Undrained soil behavior*

Skempton's pore pressure parameter B defined as the ratio of the excess pore-water pressure to the increment of total isotropic stress in undrained conditions, also calculated



$$B = \frac{\Delta p_w}{\Delta \sigma} = \frac{1}{1 + n \cdot \dfrac{\beta_w \left(1 + \dfrac{1-S}{p_w \beta_w}\right) - 1}{\beta}}$$

Predicted variation of the B-parameter with the strain during tests

Conclusions - Soil behavior in isotropic compression

The compression induced liquefaction is modeled within the frame of a continuum mixture theory for a three-phase elastoplastic porous medium. This model was developed for the investigation of the susceptibility of sands to liquefaction in undrained isotropic compression loading conditions.

Undrained loading conditions prevail in a good approximation when the characteristic loading time is relatively small, such as in cases of explosions and earthquakes (i.e. 0.001÷0.1 s).

This model is subsequently used for the analysis of the experimental results of Fragaszy and Voss (1984). n these experimental tests, the initial dry density of the sand, the initial effective stress state, and the magnitude of the total stress cycle were systematically varied to determine the influence of each on liquefaction potential.

The model is in agreement with the experiments which show that the most critical parameters affecting sand’s susceptibility to liquefaction is its density and the initial confining pressure.

References

Biot M.A., “Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range”, *The Journal of the Acoustical Society of America*, Vol. 28, No. 2, 1956, p. 168-178.

Bowen R.M., *Theory of mixtures*. In Continuum Physics, Vol.III-Mixtures and EM Field Theories (C. Eringen ed.), Academic Press, New York, 1978, p. 1-127.

Fragaszy R.J., Voss M.E., “Undrained isotropic compression behavior of Eniwetok and Monterey sands”, AFWL-TR-83-126, Final Report, San Diego State University Foundation, 1984.

Truesdell C., Toupin R., *The Classical Field Theories*, Encyclopedia of Physics, Vol. III/1 Principles of Classical Mechanics and Field Theory, Springer-Verlag, Berlin, 1960.

Vardoulakis I., Beskos D.E., “Dynamic behavior of nearly saturated porous media”, *Mechanics of Materials* 5, 1986, p. 87-108.

Vardoulakis I., Sulem J., *Bifurcation Analysis in Geomechanics*, Blackie Academic and Professional, 1995.