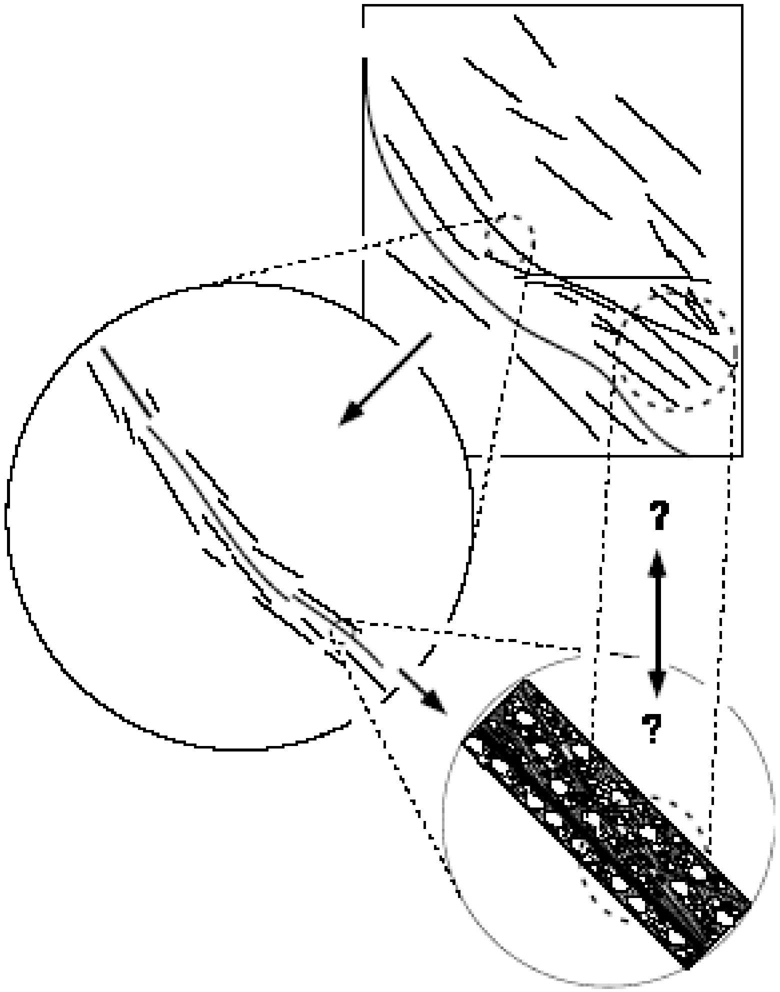
Part 2

Effect of fluid and temperature

Fault structures at different scales



(Ben Zion & Sammis, 2003)

• Regional scale (1 à 100 km)

Network of oriented fractures of the earth crust

• Local scale (1 à 100m)

Individual major faults in the network consist in an array of parallel deformation bands containing zones of intense fracturing surrounded by damaged zones

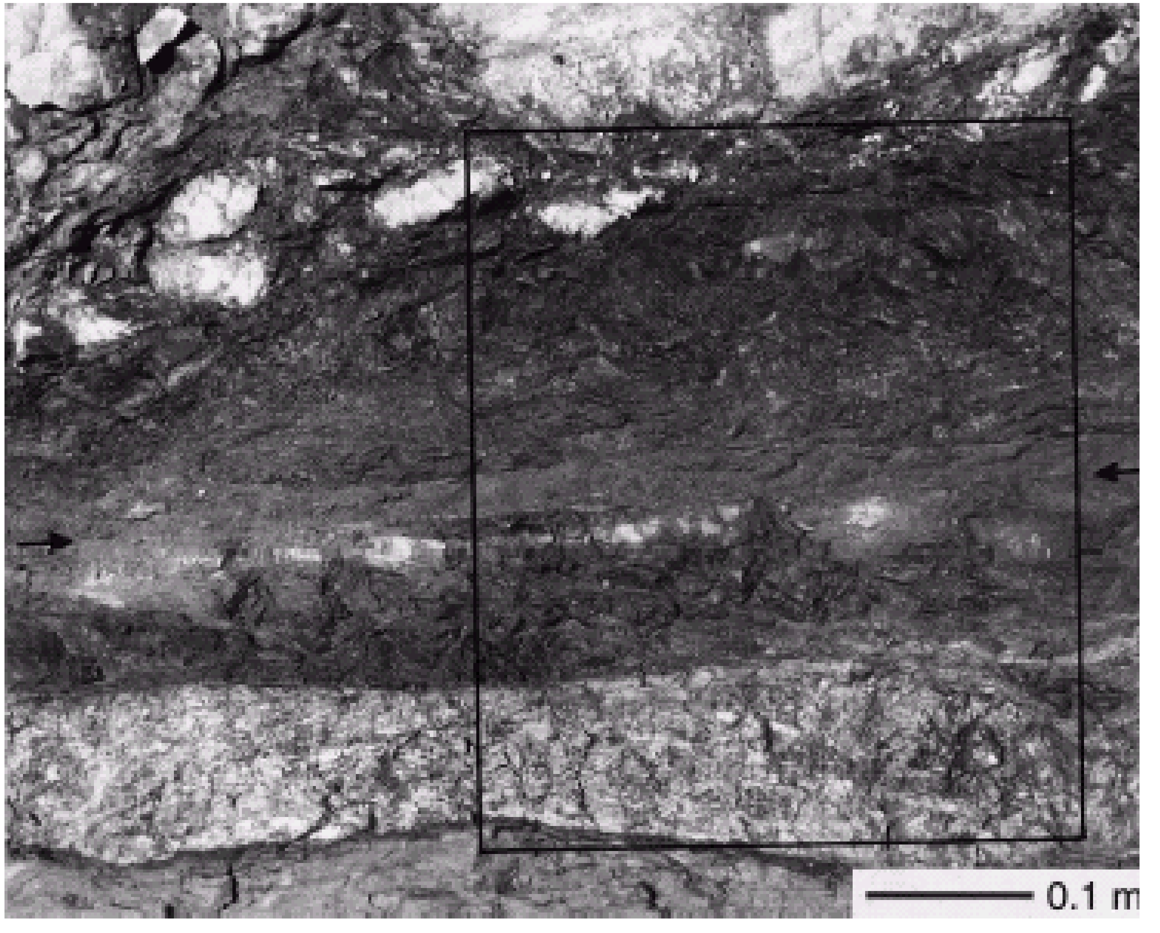
• Small scale (0.1 à 10 cm)

Shear band characterized by intense fracturation (ultra-cataclasite) and **strain localisation in very narrow slip zones (<1mm)**

Strain localisation in faults

Chester & Chester, (1998)
Punchbowl Fault
prominent slip
surface

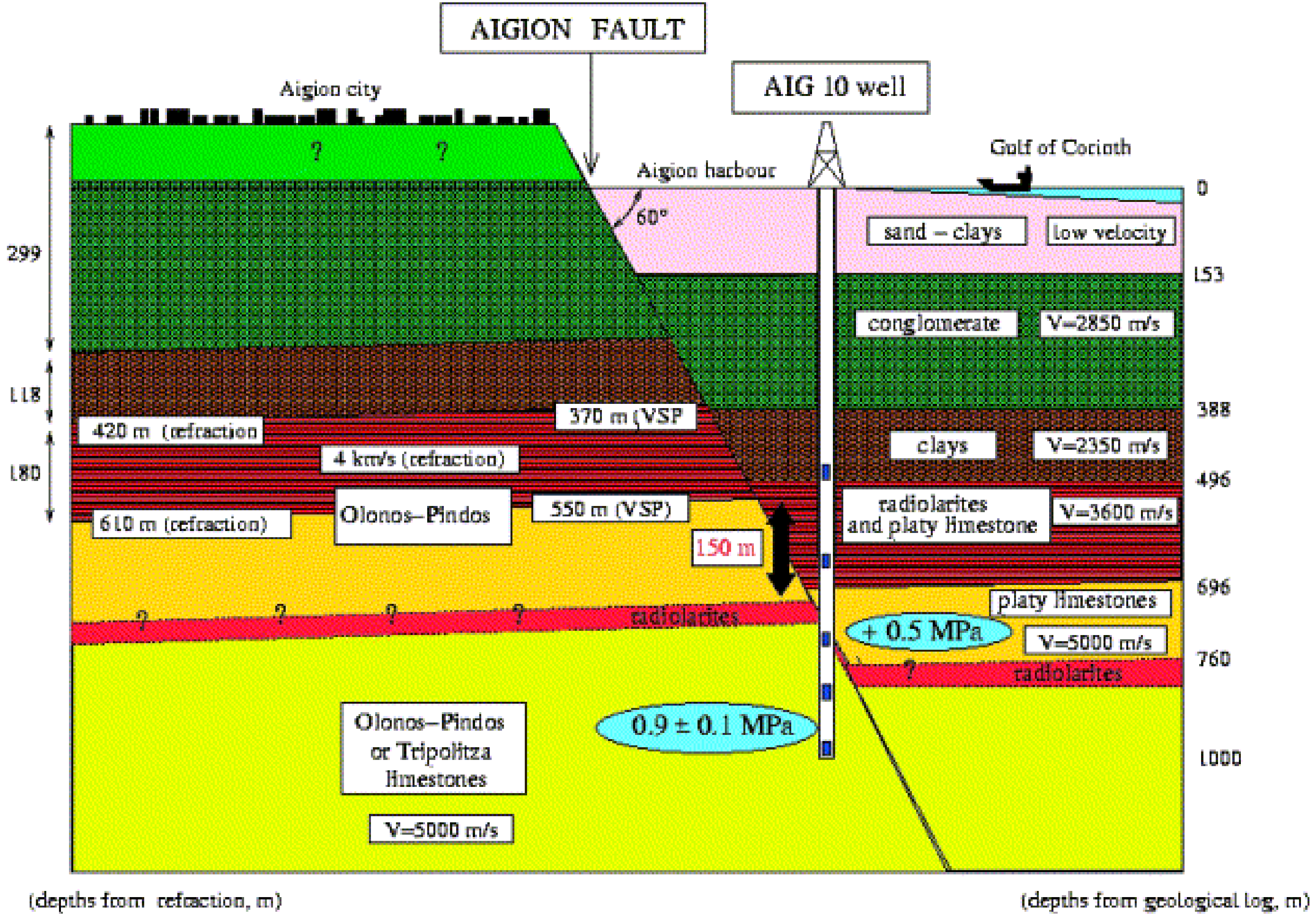
F.M. Chester, J.S. Chester / Tectonophysics 295 (1998) 199–221



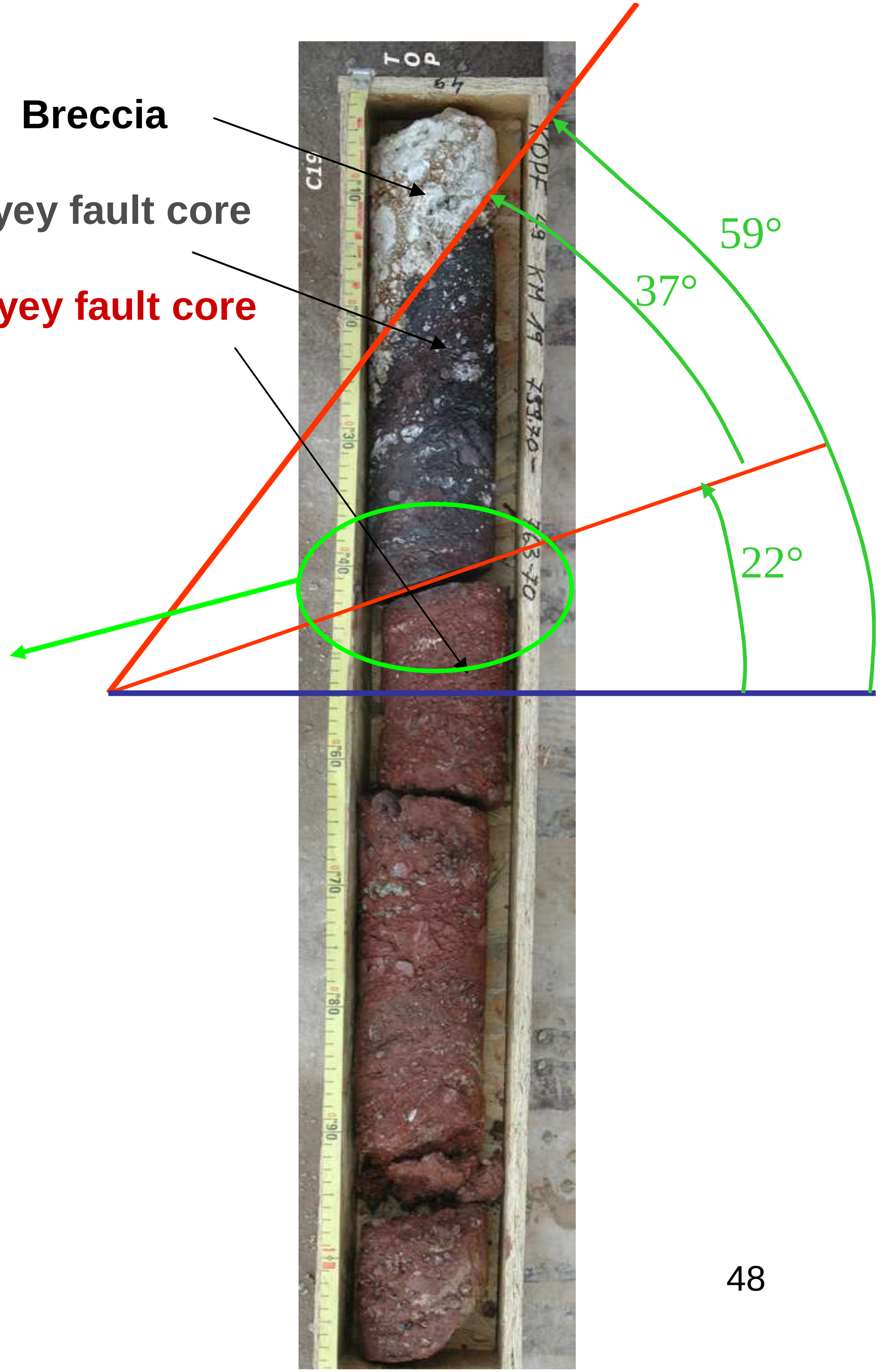
« The core of the fault consists of an ultracataclasite layer of shear localization with a thickness of cm to tens of cm containing a single planar slip surface that accommodated several tens of km of slip ».

Schematic structural cross section through Aigion Fault, in Aigion harbour

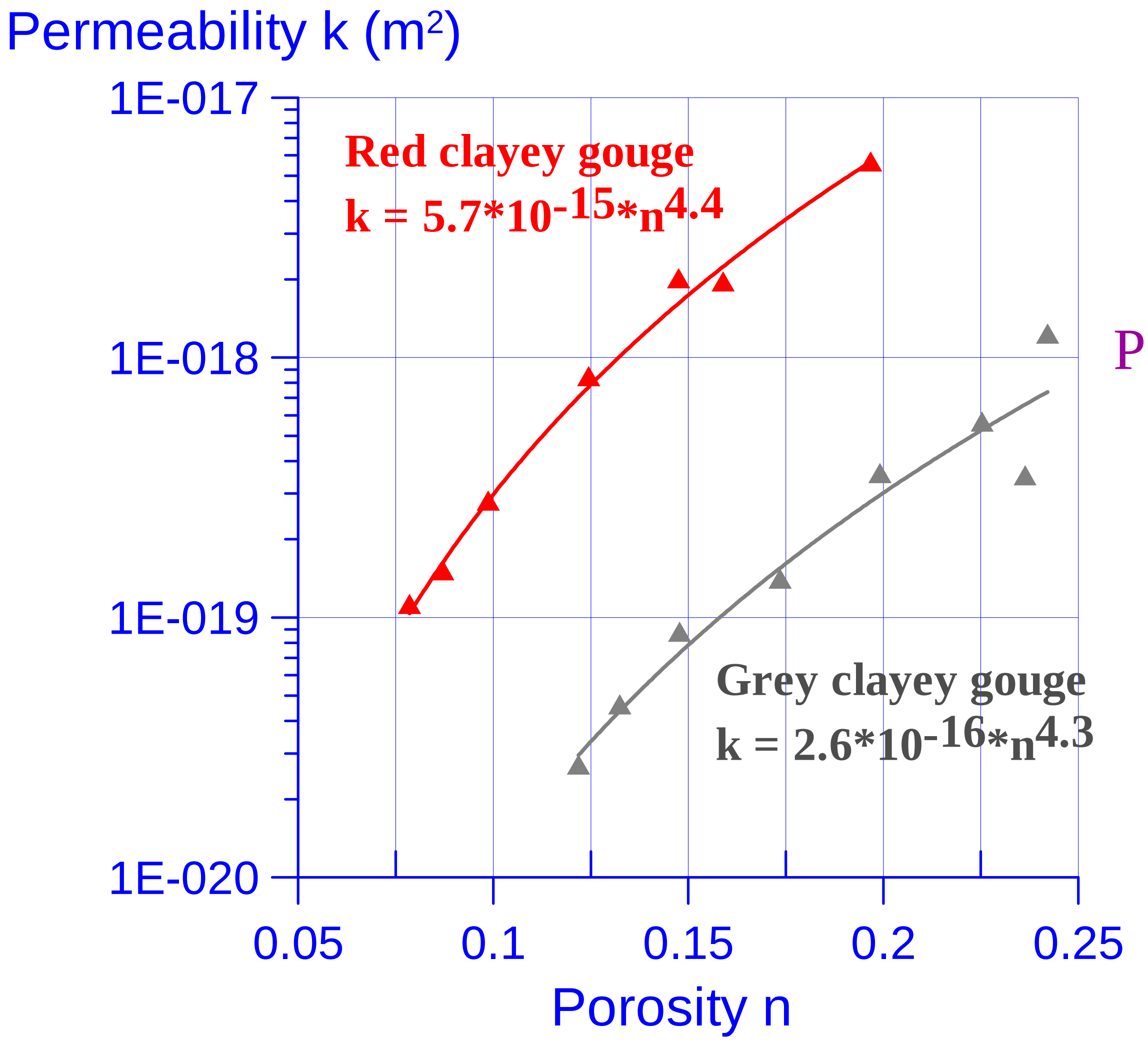
Cornet et al, 2004: *Comptes Rendus Geosciences*, **336**, 4-5, 395-406



Slip plane in the clayey gouge of Aigion fault at 760m depth



Permeability of Aigion fault clayey gouge



Permeability : 10^{-18} à 10^{-19} m^2

(10^{-3} à 10^{-4} md)

Sulem et al (2005)

Soils and Foundations 45(2), 97-108

Field observations

•Earthquakes are the result of a *frictional instability* and occur by sudden slippage along a pre-existing fault.

•Earthquakes occur because the frictional resistance to slip on a fault *weakens* with increasing slip or slip rate.

• Seismic slip is a *dynamic process* with slip rates of the order of 1m/s.

• Slip in individual earthquakes is extremely localized inside a *very thin slip zone* of few millimeters thick, thus *thermal effects* are of primary importance.

• Fault zones commonly exhibit the *presence of fluid* interacting with the rock.

Effect of pore fluid pressure and temperature during seismic or gravitational slip

Thermal pressurization of pore fluids

- The permeability of the highly granulated fault gouge is very low.
- Fluids and heat are trapped inside the slip zone during an earthquake

Thermal pressurization of the fluid occurs because the thermal expansion coefficient of water is much greater than that of the rock particles.

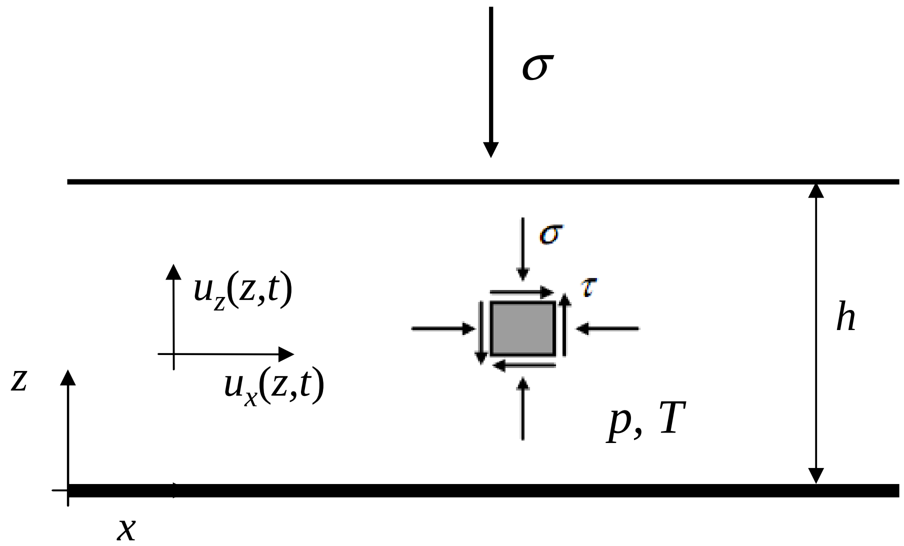
Shear heating is a mechanism of shear strength weakening because thermal pressurization of pore fluid reduces the effective stress

(*Lachenbruch, 1980, Vardoulakis, 2002, Sulem et al. 2005, Rice 2006, Ghabezloo & Sulem, 2009*).

Thermal pore fluid pressurization also occurs in large landslides (Vardoulakis, 2002, lecture of M. Veveakis on Saturday)

Undrained adiabatic shearing of a saturated rock layer

Destabilizing effect of shear heating and pore fluid pressurization



Shear strain and volume strain

$$\gamma = \frac{\partial u_x}{\partial z} \quad \varepsilon = \frac{\partial u_z}{\partial z}$$

Uniform state of stress in the layer

$$\frac{\partial \tau}{\partial z} = 0 \quad \frac{\partial \sigma}{\partial z} = 0$$

Thermo-elasto-plastic constitutive equations

$$\dot{\gamma} = \frac{1}{G}\dot{\tau} + \dot{\gamma}^p \; ; \dot{\varepsilon} = \frac{1}{K}(\dot{\sigma} - b\dot{p}) - \alpha_s \dot{T} + \dot{\varepsilon}^p$$

b : Biot coefficient, $b = 1 - \frac{K}{K_s}$, K_s bulk modulus of the solid matrix

α_s : thermal dilation coefficient of the empty porous solid.

Fluid mass balance

Fluid mass per unit volume of porous medium $m_f = \rho_f n$

n is the pore volume fraction (Lagrangian porosity)

ρ_f and is the density of the saturating fluid.

$$\frac{\partial m_f}{\partial t} = -\frac{\partial q_f}{\partial z}$$

q_f is the flux of fluid

$$\frac{\partial m_f}{\partial t} = n\frac{\partial \rho_f}{\partial t} + \rho_f \frac{\partial n}{\partial t}$$

$$q_f = -\frac{\rho_f}{\eta_f} k_f \frac{\partial P_p}{\partial z}$$

Darcy law for the fluid flow, with viscosity η_f through a material with permeability k_f

$$\frac{\partial \rho_f}{\partial t} = \rho_f \beta_f \frac{\partial P_p}{\partial t} - \rho_f \lambda_f \frac{\partial T}{\partial t}$$

$$\frac{\partial n}{\partial t} = n\beta_n \frac{\partial p}{\partial t} + n\lambda_n \frac{\partial T}{\partial t} + \frac{\partial n^p}{\partial t}$$

$\frac{\partial n^p}{\partial t}$: rate of plastic porosity change

$\beta_f = \frac{1}{\rho_f}\left(\frac{\partial \rho_f}{\partial P_p}\right)_T$: pore fluid compressibility

$\lambda_f = -\frac{1}{\rho_f}\left(\frac{\partial \rho_f}{T}\right)_{P_p}$: pore fluid thermal expansion coefficient

$\beta_n = \frac{1}{n}\left(\frac{\partial n}{\partial P_p}\right)_T$: pore volume compressibility $\beta_n = \frac{1}{n}\left(\beta_d - (1+n)\beta_s\right)$

$\lambda_n = \frac{1}{n}\left(\frac{\partial n}{\partial T}\right)_{P_p}$: pore volume thermal expansion coefficient

β_d : compressibility of the porous rock

β_s: compressibility of the solid phase

Pore fluid production and diffusion equation:

$$\frac{\partial p}{\partial t} = c_{hy}\frac{\partial^2 p}{\partial z^2} + \Lambda\frac{\partial T}{\partial t} - \frac{1}{\beta^*}\frac{\partial n^p}{\partial t}$$

$$\Lambda = \frac{\lambda_f - \lambda_n}{\beta_n + \beta_f}$$

is the coefficient of thermal pressurization

(typical values: 0.1 to 1 MPa/℃)

$\beta^* = n\left(\beta_n + \beta_f\right)$ is the storage coefficient.

For incompressible fluid and solid phase $\beta^* = 1/K$

$c_{hy} = k_f/(\beta\eta_f)$ is the hydraulic diffusivity

For plastically incompressible solid matrix: $$\frac{\partial n^p}{\partial t} = \frac{\partial \varepsilon^p}{\partial t}$$

Energy balance equation

$$\rho C \frac{\partial T}{\partial t} = E_F - \frac{\partial q_h}{\partial z}$$

E_F is the rate of frictional heat produced during slip

ρC is the specific heat per unit volume of the fault material

q_h is the heat flux and according to the Fourier law is proportional to the temperature gradient

$$q_h = -k_T \frac{\partial T}{\partial z}$$

k_T is the thermal conductivity of the saturated rock

It is assumed that all the plastic work is converted into heat

$$E_F = \tau \dot{\gamma}^p$$

$$\frac{\partial T}{\partial t} = c_{th} \frac{\partial^2 T}{\partial z^2} + \frac{1}{\rho C} \tau \dot{\gamma}^p$$

$c_{th} = k_T / \rho C$ is the thermal diffusivity

Undrained adiabatic limit

The drainage and the heat flux are prohibited at the boundaries of the layer.

$$q_f = 0 \text{ and } q_h = 0$$

The normal stress σ acting on the sheared layer is constant.

$$\dot{\sigma} = 0$$

The undrained adiabatic limit is applicable as soon as the slip event is sufficiently rapid and the shear zone broad enough to effectively preclude heat or fluid transfer (e.g. earthquakes, landslides).

$$\dot{p} = \Lambda \dot{T} - \frac{1}{\beta^*} \dot{\varepsilon}^p$$

$$\dot{T} = \frac{1}{\rho C} \tau \dot{\gamma}^p$$

$$\dot{\gamma}^p = \frac{1}{H}\left(\dot{\tau} - \mu\left(\dot{\sigma} - \dot{p}\right)\right)$$

$$\dot{\varepsilon}^p = \beta \dot{\gamma}^p$$

Undrained adiabatic limit

$$\dot{p} = \frac{\frac{\Lambda\tau}{\rho C} - \frac{\beta}{\beta^*}}{H - \mu\left(\frac{\Lambda\tau}{\rho C} - \frac{\beta}{\beta^*}\right)}\dot{\tau}$$

$$\dot{\gamma} = \left(\frac{1}{G} + \frac{1}{H + \mu\beta/\beta^* - \mu\Lambda\tau/\rho C}\right)\dot{\tau}$$

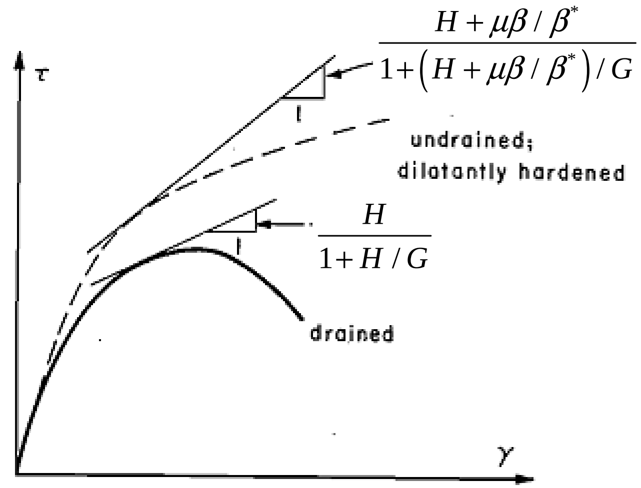
$$\dot{\tau} = \left(\frac{H + \mu\beta/\beta^* - \mu\Lambda\tau/\rho C}{1 + \left(H + \mu\beta/\beta^* - \mu\Lambda\tau/\rho C\right)/G}\right)\dot{\gamma}$$

For incompressible fluid and solid phase $\beta^* = 1/K$

Without thermal effects $(\Lambda = 0)$ (Rice 1975)

- Drained response: $\dot{\tau} = \dfrac{H}{1 + H/G}\dot{\gamma}$

- Undrained response: $\dot{\tau} = \dfrac{H + \mu\beta/\beta^*}{1 + \left(H + \mu\beta/\beta^*\right)/G}\dot{\gamma}$



Undrained adiabatic limit

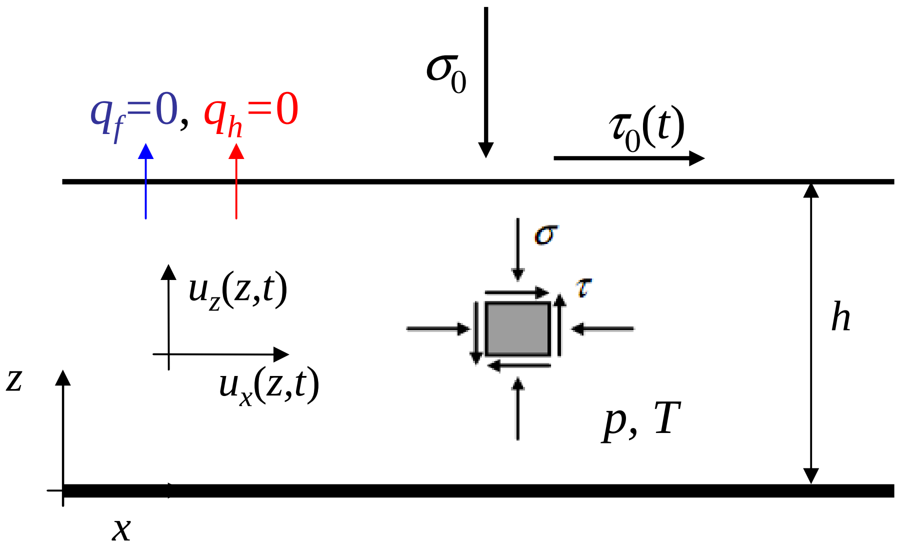
$$\dot{\tau} = \left(\frac{H + \mu\beta / \beta^* - \mu\Lambda\tau / \rho C}{1 + \left(H + \mu\beta / \beta^* - \mu\Lambda\tau / \rho C \right) / G} \right) \dot{\gamma}$$

Dilatant hardening effect in undrained conditions

Thermal softening effect in adiabatic conditions (thermal pore fluid pressurization)

When thermal softening prevails against strain hardening, plastic localization in the form of adiabatic shear banding can occur

Stability analysis of the undrained adiabatic limit



The layer is sheared by application of a monotonically increasing shear stress to its boundary while the normal stress is held constant

Drainage and heat fluxes are prevented at the boundaries. However, the internal fluid and heat flows are permitted inside the layer.

Homogeneous solution:

$$\tau_0,\, \gamma_0,\, \varepsilon_0,\, p_0,\, T_0$$

$$\gamma = \gamma_0 + \tilde{\gamma}\,;\, \varepsilon = \varepsilon_0 + \tilde{\varepsilon}\,;\, \sigma = \sigma_0 + \tilde{\sigma}\,;\, \tau = \tau_0 + \tilde{\tau}\,;\, p = p_0 + \tilde{p}\,;\, T = T_0 + \tilde{T}$$

Perturbation quantitities $\tilde{\gamma}$, $\tilde{\tau}$ etc...

Equilibrium equations:

$$\frac{\partial \tau}{\partial z} = 0, \quad \frac{\partial \sigma}{\partial z} = 0 \Rightarrow \tau = \tau_0,\, \sigma = \sigma_0,\, \tilde{\tau} = 0,\, \tilde{\sigma} = 0$$

The stress field is uniform within the layer

Stability analysis of the undrained adiabatic limit

Balance equations: Rate governing equations for the perturbation quantities

$$\frac{\partial \tilde{p}}{\partial t} = c_{hy} \frac{\partial^2 \tilde{p}}{\partial z^2} + \Lambda \frac{\partial \tilde{T}}{\partial t} - \frac{1}{\beta^*} \frac{\partial \tilde{\varepsilon}^p}{\partial t}$$

$$\frac{\partial \tilde{T}}{\partial t} = c_{th} \frac{\partial^2 \tilde{T}}{\partial z^2} + \frac{1}{\rho C} \tau_0 \frac{\partial \tilde{\gamma}^p}{\partial t}$$

Constitutive equations

$$\frac{\partial \tilde{\gamma}^p}{\partial t} = \frac{\mu}{H} \frac{\partial \tilde{p}}{\partial t}$$

$$\frac{\partial \tilde{\varepsilon}^p}{\partial t} = \beta \frac{\partial \tilde{\gamma}^p}{\partial t}$$

Stability analysis of the undrained adiabatic limit

We introduce the constitutive in the balance equations for the perturbation quantities

$$\left(1+\frac{\beta}{\beta^*}\frac{\mu}{H}\right)\frac{\partial \tilde{p}}{\partial t}-\Lambda\frac{\partial \tilde{T}}{\partial t}-c_{hy}\frac{\partial^2 \tilde{p}}{\partial z^2}=0$$

$$\frac{\partial \tilde{T}}{\partial t}-\frac{1}{\rho C}\tau_0\frac{\mu}{H}\frac{\partial \tilde{p}}{\partial t}-c_{th}\frac{\partial^2 \tilde{T}}{\partial z^2}=0$$

The spatial dependence of the perturbations is decomposed into Fourier modes with wavelenth λ:

$$\tilde{p}=P(0)e^{st}\cos\left(\frac{2\pi z}{\lambda}\right);\tilde{T}=T(0)e^{st}\cos\left(\frac{2\pi z}{\lambda}\right)$$

With $\lambda = h/n$, n is an integer, the zero heat and fluid flux boundary conditions at $y = 0$, h are satisfied.

s is the growth coefficient in time of the perturbation

$$\text{if } Re(s)\geq 0, \text{instability}; \text{ if } Re(s)<0, \text{stability}$$

Stability analysis of the undrained adiabatic limit

$$\left(\left(1+\frac{\beta}{\beta^*}\frac{\mu}{H}\right)s+c_{hy}\left(\frac{2\pi}{\lambda}\right)^2\right)P(0)-\Lambda sT(0)=0$$

$$-\frac{\mu}{\rho CH}\tau_0 sP(0)+\left(s+c_{hy}\left(\frac{2\pi}{\lambda}\right)^2\right)T(0)=0$$

For non trivial solutions

$$\det\begin{pmatrix} s\left(1+\frac{\beta\mu}{\beta^* H}\right)+c_{hy}\left(\frac{2\pi}{\lambda}\right)^2 & -\Lambda s \\ -\frac{\mu}{\rho CH}\tau_0 s & \left(c_{th}\left(\frac{2\pi}{\lambda}\right)^2+s\right) \end{pmatrix}=0$$

$$\left(\frac{\Lambda\mu}{\rho C}\tau_0-\frac{\beta\mu}{\beta^*}-H\right)\left(\frac{\lambda}{2\pi}\right)^4 s^2-\left(\left(c_{th}+c_{hy}\right)H+\frac{\beta\mu}{\beta^*}c_{th}\right)\left(\frac{\lambda}{2\pi}\right)^2 s-c_{th}c_{hy}H=0$$

Stability analysis of the undrained adiabatic limit

Quadratic equation for the growth coefficient s

$$\left(H+\frac{\beta\mu}{\beta^*}-\frac{\Lambda\mu}{\rho C}\tau_0\right)\left(\frac{\lambda}{2\pi}\right)^4 s^2+\left(\left(c_{th}+c_{hy}\right)H+\frac{\beta\mu}{\beta^*}c_{th}\right)\left(\frac{\lambda}{2\pi}\right)^2 s+c_{th}c_{hy}H=0$$

If a solution has a positive real part, then the corresponding perturbation grows exponentially in time.

Without the thermal effect, i.e. $\Lambda = 0$, $c_{th} = 0$

$$s=-\left(\frac{2\pi}{\lambda}\right)^2\frac{c_{hy}H}{H+\dfrac{\beta\mu}{\beta^*}}$$

Stability condition: $H > 0$

Undrained shearing is stable only in those circumstances for which the underlying drained deformation would be stable (Rice 1975).

Stability analysis of the undrained adiabatic limit

Stability condition with thermal effects:

$$H > 0 \text{ and } \frac{\Lambda\mu}{\rho C}\tau_0 < \frac{\beta\mu}{\beta^*} + H$$

- The system is always *unstable in the softening regime*.

- The stability condition for *undrained adiabatic shearing* is more restrictive than the one for *undrained shearing*.

- This result demonstrates the *destabilizing effect of thermal fluid pressurization*: undrained adiabatic shearing of a material with positive strain hardening is stable only when the thermal pressurization is not too high.

Ill-posedness of undrained shearing in the softening regime

Without the thermal effect, i.e. $\Lambda = 0$, $c_{th} = 0$

$$s = -\left(\frac{2\pi}{\lambda}\right)^2 \frac{c_{hy} H}{H + \dfrac{\beta\mu}{\beta^*}}$$

Instability occurs at the same time for all wave lengths.

The growth coefficient is infinite for the infinitely small wave length limit (ill-posedness).

In the softening regime, the instability tends to localize in a strip of zero thickness. The classical theory is unable to simulate the localization of the deformation in a shear band of finite thickness.

Ill-posedness of undrained shearing in the softening regime

Pore pressure evolution without thermal effect ($\Lambda=0$)

$$\left(1+\frac{\beta}{\beta^{*}}\frac{\mu}{H}\right)\frac{\partial \tilde{p}}{\partial t}-c_{hy}\frac{\partial^{2}\tilde{p}}{\partial z^{2}}=0$$

$$\frac{\partial \tilde{p}}{\partial t}=c_{hy}\frac{H}{H+\dfrac{\beta}{\beta^{*}}\mu}\frac{\partial^{2}\tilde{p}}{\partial z^{2}}$$

For *H<0*, Diffusion equation with negative diffusivity

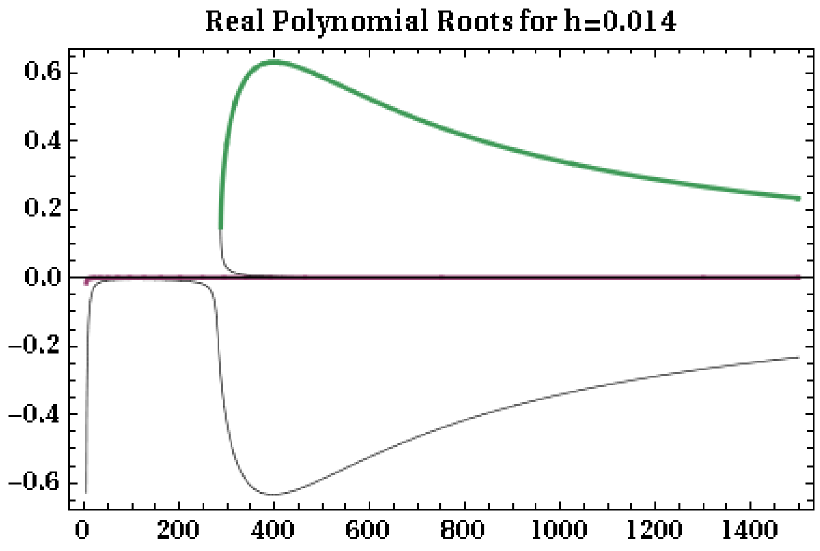
Mathematically ill-posed *backward-in-time* ‘antidiffusion’ equation (Vardoulakis, 1996)

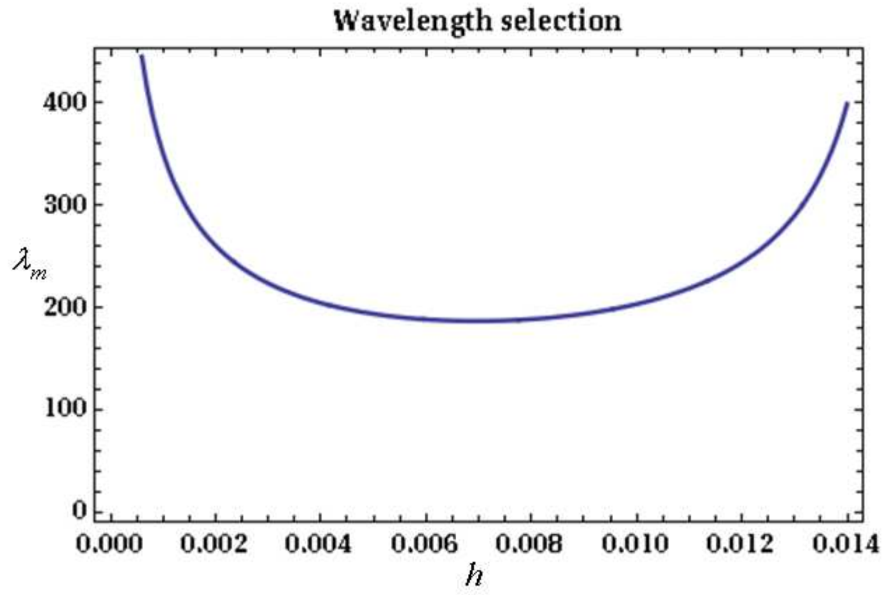
Regularization of Ill-posedness with inertia terms and microstructure

To regularize the ill-posed problem it is necessary to resort to continuum models with *microstructure* to describe correctly localization phenomena (eg. Cosserat continuum, second grade continuum).

The layer is modeled as a Cosserat Continuum. The characteristic equation for the growth coefficient *s* is a polynomial of degree 8.

Wave length selection is obtained (*Sulem, Stefanou, Veveakis, 2010*)





CONCLUSIONS Part 2

- Shear heating and pore fluid pressurization have a destabilizing effect on shearing of a saturated rock layer

- Instability can occur even in the hardening regime of the underlying drained stress strain response if dilatant hardening cannot compensate the thermal pressurization of the pore fluid.

- If the effect of microstructure and micro-inertia are not considered, the underlying mathematical problem is ill-posed, i.e. for a hardening modulus lower than the critical hardening modulus at instability, the growth coefficient of the instability is infinite.

- The complete dynamic analysis for a Cosserat continuum leads to a wave length selection of the instability mode as the growth coefficient in time is maximum and finite for a particular wave length.